Le nozze di Giustizia

Interactions between Artificial Intelligence, Logic, Law, Language and Computation

Joost J. Joosten

Monday, September 18, 2023

Many thanks to the organisers

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- Link of my talk to

may go through Article 15 on Robustness

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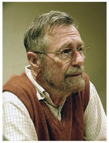
    - Adaptive to environment;
    - Computation;
    - Adequate responses;

Then, what is Artificial



- External to humans

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  Can machines think?
- Edsger Wybe Dijkstra: The question whether a machine can think is as relevant as the question if a submarine can swim.

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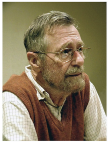
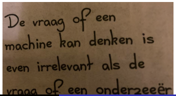


- External to humans
- EWD:



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- Unless we are happy with ChatGPT-like answers and dialogues as being authoritative (which is not entirely ridiculous)

Should we use it?

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- St. Augustine vs Ibn Taymiyya, Al Ghazali and, Nizam al-Mulk

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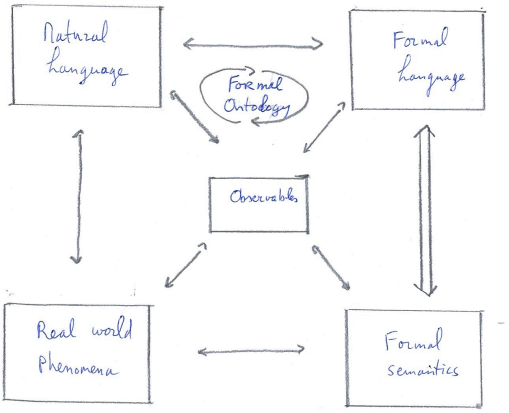
- Formal versus real: Formal Language, Formal Semantics, Informal phenomenon (language and semantics)
- Limited scope (no discretionary powers, clear ontologies)



Formal methods I have been involved with

1. Logical, mathematical and algorithmic analysis of legal texts

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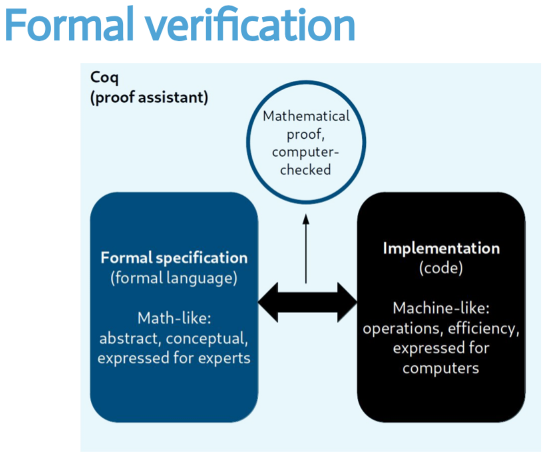
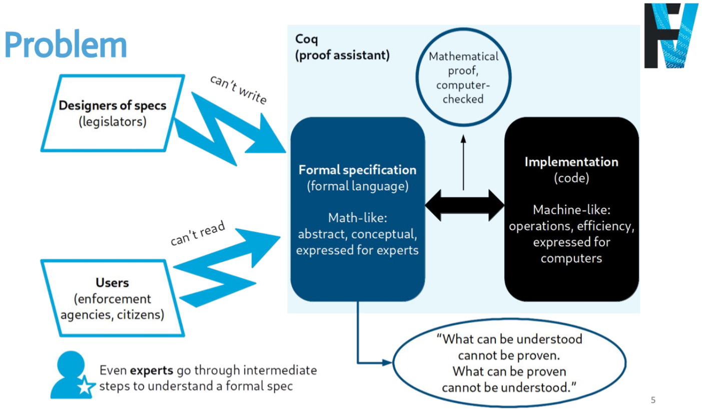
3. Model checking

Program correctness



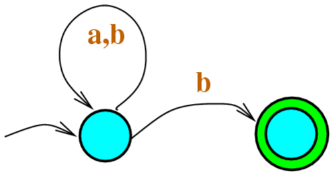
Formal and technical specification


Problem
Designers of specs
(legislators)
can't write
Users
(enforcement agencies, citizens)
can't read
Even experts go through intermediate steps to understand a formal spec
Coq
(proof assistant)
Mathematical proof, computer-checked
Formal specification
(formal language)
Math-like: abstract, conceptual, expressed for experts
Implementation
(code)
Machine-like: operations, efficiency, expressed for computers
"What can be understood cannot be proven. What can be proven cannot be understood."
5

Model checking

You must end your day with activity *b*:

Finite automata



$\Sigma = \{a,b\}$

$L = (a+b)^*b$

NFA

Limitations

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- First and most famous example: Halting Problem

Simple halting program

- Given a program Π, does it hold on input A?

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    otherwise (that is, if $\$x = 0$) it will HALT.

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  - $\$x = 2$;
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  - $\$x = 1;$
  - HALT.

Reductio ad absurdum

- Suppose there existed a program $H(X, Y)$ with the property

$$H(X, Y) = \begin{cases} 1 & \text{if } X(Y)\downarrow \\ 0 & \text{if } X(Y)\uparrow \end{cases}$$

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- $\lightning$

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- etc.

Devastating consequences of Turing

- There exists no infallible virus scanner
- with or without AI
- Unrestricted program verification is impossible
- with or without AI
- etc.
- Many problems are just not solvable in the mere virtue of logic (with our without AI)

Just divorce

- Imagine a *Just divorce* law that say:

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  Upon divorce, each item in the common patrimony should be assigned a € amount and then the patrimony should be split between the partners in such a way that causes the minimal € difference in the division of goods between the partners

Infeasible problems

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10715086071862673209484250490600018105614048117055336
07443750388370351051124936122493198378815695858127594
67291755314682518714528569231404359845775746985748039
34567774824230985421074605062371141877954182153046474
98358194126739876755916554394607706291457119647768654
2167660429831652624386837205668069376

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$$\begin{gathered}1071508607186267320948425049060001810561404811705 5336\\ 0744375038837035105112493612249319837881569585812759 4\\ 6729175531468251871452856923140435984577574698574803 9\\ 3456777482423098542107460506237114187795418215304647 4\\ 9835819412673987675591655439460770629145711964776865 4\\ 2167660429831652624386837205668069376\end{gathered}$$

- With the best supercomputers, this would take more than the estimated age of the universe
- not a practical law

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- but are unfeasible
- Legal stipulations and computable laws should avoid those

Bad mathematical behaviour

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- Our team is looking for new similar laws: please do share suggestions with me

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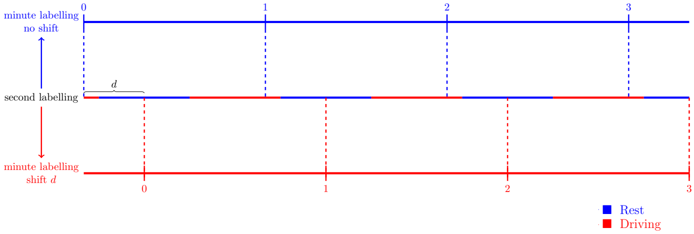

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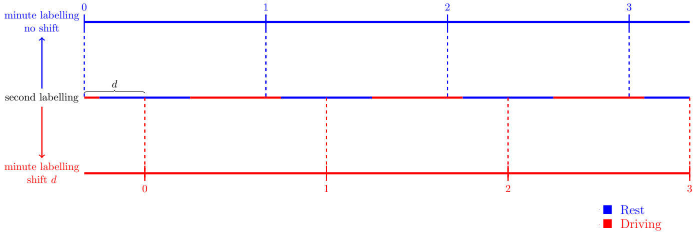


- Ignoring UTC makes this a relevant issue!!!

Hidden dynamics

Regulation (EU) 2016/799:

(51) Given a calendar minute, if DRIVING is registered as the activity of both the immediately preceding and the immediately succeeding minute, the whole minute shall be regarded as DRIVING.

(52) Given a calendar minute that is not regarded as DRIVING according to requirement 051, the whole minute shall be regarded to be of the same type of activity as the longest continuous activity within the minute (or the latest of the equally long activities).

More hidden dynamics

§4(h) ‘regular weekly rest period’ means any period of rest of at least 45 hours.

§4(i) ‘a week’ means the period of time between 00.00 on a Monday and 24.00 on the following Sunday.

§8.6. In any two consecutive weeks, a driver shall take at least:

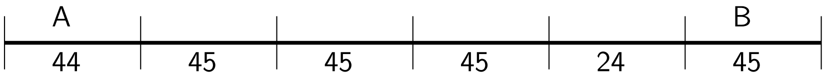
- two regular weekly rest periods, or
- one regular weekly rest period and one reduced weekly rest period of at least 24 hours. However, the reduction shall be compensated by an equivalent period of rest taken en bloc before the end of the third week following the week in question.

A weekly rest period shall start no later than at the end of six 24-hour periods from the end of the previous weekly rest period.

§8.7. Any rest taken as compensation for a reduced weekly rest period shall be attached to another rest period of at least nine hours.

§8.9. A weekly rest period that falls in two weeks may be counted in either week, but not in both.

More hideous behaviour



Illegal

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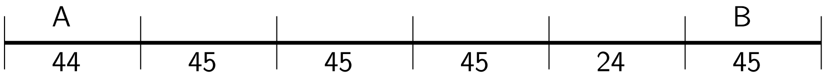

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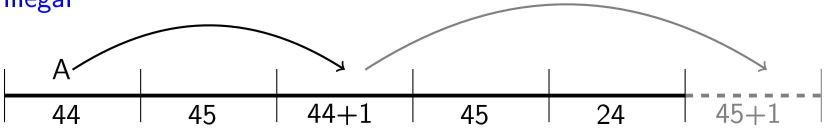

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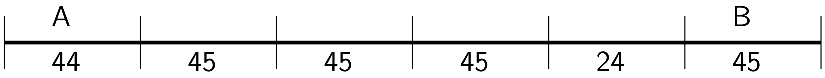

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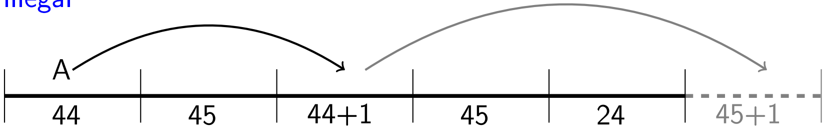

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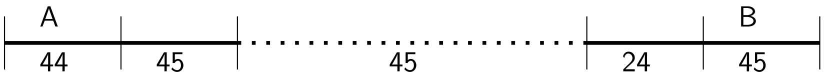
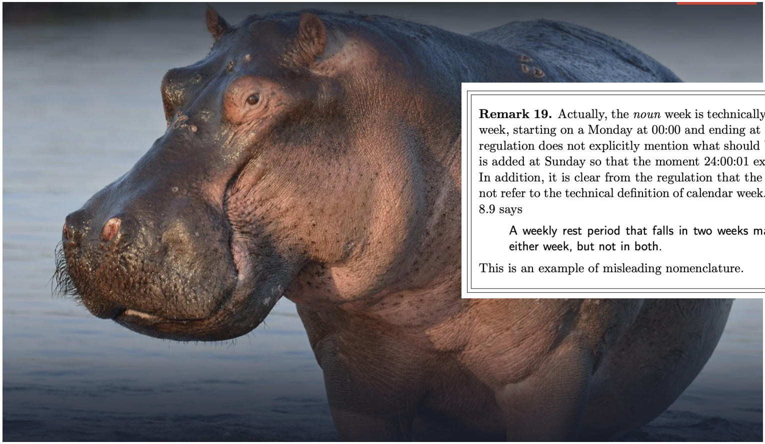
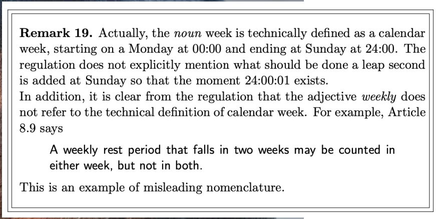


This can be iterated indefinitely: non-locality

Bad formal ontology





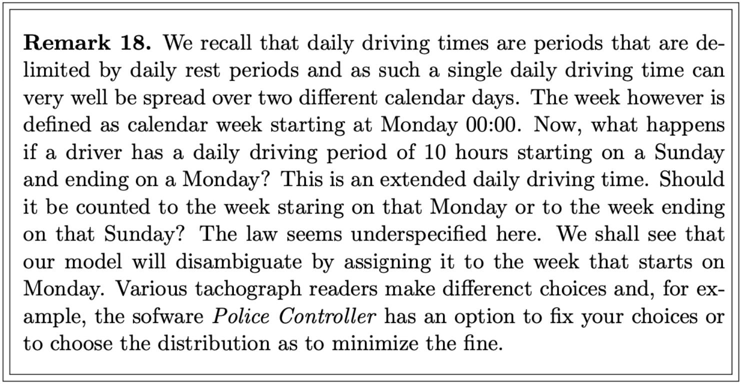
Remark 19. Actually, the *noun* week is technically defined as a calendar week, starting on a Monday at 00:00 and ending at Sunday at 24:00. The regulation does not explicitly mention what should be done a leap second is added at Sunday so that the moment 24:00:01 exists.
In addition, it is clear from the regulation that the adjective *weekly* does not refer to the technical definition of calendar week. For example, Article 8.9 says

> A weekly rest period that falls in two weeks may be counted in either week, but not in both.

This is an example of misleading nomenclature.

Programmer as judge

Article 6.1: The daily driving time shall not exceed nine hours. However, the daily driving time may be extended to at most 10 hours not more than twice during the week.

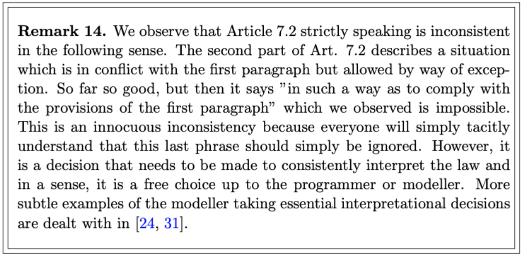


Remark 18. We recall that daily driving times are periods that are delimited by daily rest periods and as such a single daily driving time can very well be spread over two different calendar days. The week however is defined as calendar week starting at Monday 00:00. Now, what happens if a driver has a daily driving period of 10 hours starting on a Sunday and ending on a Monday? This is an extended daily driving time. Should it be counted to the week staring on that Monday or to the week ending on that Sunday? The law seems underspecified here. We shall see that our model will disambiguate by assigning it to the week that starts on Monday. Various tachograph readers make differenct choices and, for example, the sofware *Police Controller* has an option to fix your choices or to choose the distribution as to minimize the fine.

(Potentially) inconsistent laws

Article 7 (1st part): After a driving period of four and a half hours a driver shall take an uninterrupted break of not less than 45 minutes, unless he takes a rest period.

Article 7 (2nd part): This break may be replaced by a break of at least 15 minutes followed by a break of at least 30 minutes each distributed over the period in such a way as to comply with the provisions of the first paragraph.



Remark 14. We observe that Article 7.2 strictly speaking is inconsistent in the following sense. The second part of Art. 7.2 describes a situation which is in conflict with the first paragraph but allowed by way of exception. So far so good, but then it says "in such a way as to comply with the provisions of the first paragraph" which we observed is impossible. This is an innocuous inconsistency because everyone will simply tacitly understand that this last phrase should simply be ignored. However, it is a decision that needs to be made to consistently interpret the law and in a sense, it is a free choice up to the programmer or modeller. More subtle examples of the modeller taking essential interpretational decisions are dealt with in [24, 31].

Violation of the spirit of the law

Article 7 (1st part): After a driving period of four and a half hours a driver shall take an uninterrupted break of not less than 45 minutes, unless he takes a rest period.

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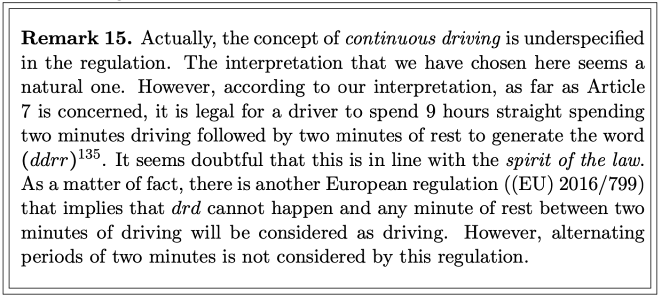

Remark 15. Actually, the concept of *continuous driving* is underspecified in the regulation. The interpretation that we have chosen here seems a natural one. However, according to our interpretation, as far as Article 7 is concerned, it is legal for a driver to spend 9 hours straight spending two minutes driving followed by two minutes of rest to generate the word $(ddrr)^{135}$. It seems doubtful that this is in line with the *spirit of the law*. As a matter of fact, there is another European regulation ((EU) 2016/799) that implies that *drd* cannot happen and any minute of rest between two minutes of driving will be considered as driving. However, alternating periods of two minutes is not considered by this regulation.

Formal ontologies not good

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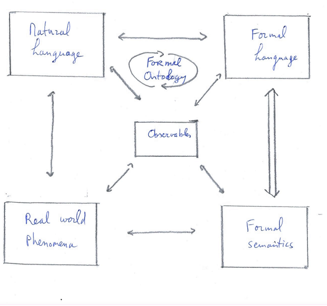

Remark 16. An important problem with the formal ontology of *continuous driving* is that it is not a physical observable like speed. Neither does it seem to be defined in an unambiguous way in terms of physical observables. Consequently we run into troubles as, for example, the one mentioned in Remark 5.1

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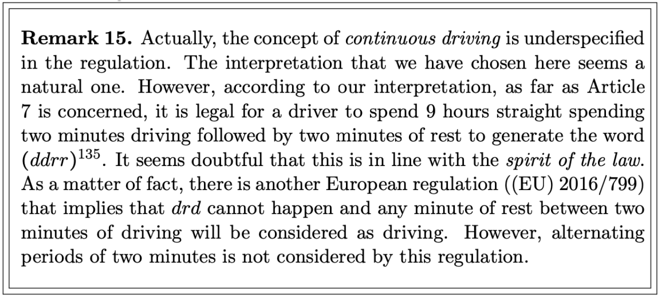

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Degenerate case

Article 7 (1st part): After a driving period of four and a half hours a driver shall take an uninterrupted break of not less than 45 minutes, unless he takes a rest period.

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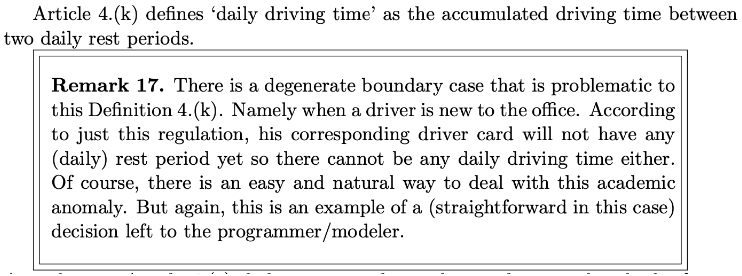

Article 4.(k) defines 'daily driving time' as the accumulated driving time between two daily rest periods.

Remark 17. There is a degenerate boundary case that is problematic to this Definition 4.(k). Namely when a driver is new to the office. According to just this regulation, his corresponding driver card will not have any (daily) rest period yet so there cannot be any daily driving time either. Of course, there is an easy and natural way to deal with this academic anomaly. But again, this is an example of a (straightforward in this case) decision left to the programmer/modeler.

CANTIERE
BUONGOVERNO

Pledge for humanities

- Cost and gdp argument

Pledge for humanities

- Cost and gdp argument
- Incorporate qualitative values into the cost argument

Preferred humanity

Those who know who and what they are do not need to ask what they should do. And those who must ask will not be able to stop asking until they begin to look inside themselves.

Joseph Weizenbaum