On Applied Logic, Controlled Natural Language and Large Language Models

Joost J. Joosten

Universitat de Barcelona

Lab42, Amsterdam Science Park
Dutch Formal Methods Day, Thursday, June 26

The business model of our research lab

Business focussed: from concrete to abstract

Work presented in collaboration with various co-authors: Moritz Müller, Juli Ponce Solé, David Fernández-Duque, Bjørn Jespersen, Ana de Almeida Borges, Eduardo Hermo Reyes, Sofia Santiago Fernández, Petia Guintech, Mireia González Bedmar, Juan Conejero Rodríguez, Marina López Chamoza, Eric Sancho Adamson, Aleix Solé Sanchez, Quim Casals Buñuel, Marta Soria Heredia, Guillermo Errezil Alberdi, Daniel Soussa E Ribeiro, etc.

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- Hence, open texture is needed
- Any automated process and in particular, any automated process in the legal sector need unambiguity
- The programmer needs to disambiguate?

An Example from a recent paper

arXiv > cs > arXiv:2503.09129

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[Submitted on 12 Mar 2025]

Specification languages for computational laws versus basic legal principles

Petia Guintchev, Joost J. Joosten, Sofia Santiago Fernández, Eric Sancho Adamson, Aleix Solé Sánchez, Marta Soria Heredia

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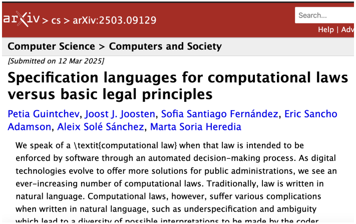
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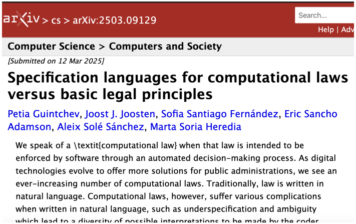
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- Article 6.1.: The daily driving time shall not exceed nine hours. However, the daily driving time may be extended to at most 10 hours not more than twice during the week.
- EU Regulation 561/2006 on road transport

Small leap in a year, giant step for a truck-driver

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- UTC and 27 leap seconds

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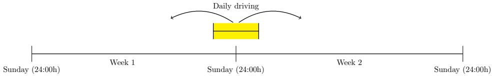

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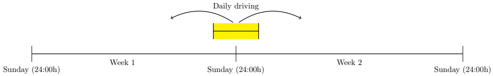

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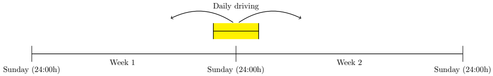

- Underspecifaction
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- Non-locality!

Non-locality in weekly rest periods

Regulation (EC) No 561/2006

§8.6. In any two consecutive weeks, a driver shall take at least:

- two regular weekly rest periods [of at least 45 hours], or
- one regular weekly rest period and one reduced weekly rest period of at least 24 hours. However, the reduction shall be compensated by an equivalent period of rest taken en bloc before the end of the third week following the week in question.

A weekly rest period shall start no later than at the end of six 24-hour periods from the end of the previous weekly rest period.

§8.9. A weekly rest period that falls in two weeks may be counted in either week, but not in both.

Let's break it down...

- Regular weekly rest: $\geq$ 45 hours

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- Any reduced rest must be compensated by a continuous block in the following three weeks

Combinatorics of rest assignments

Can we assign a week to each rest period so that each week is assigned to at least one rest period?

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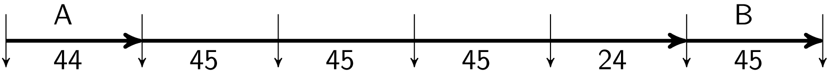
In principle this is an NP problem (assign 0 or 1 to each rest period according to whether it should belong to the earlier week or the later week).

Non-locality of compensations



Illegal

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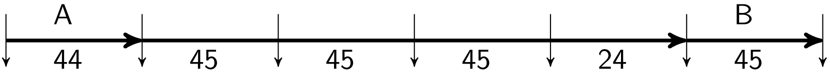

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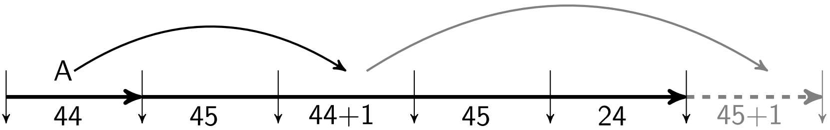

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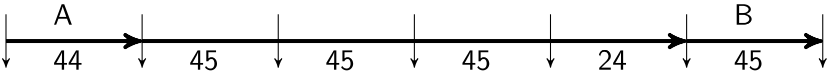

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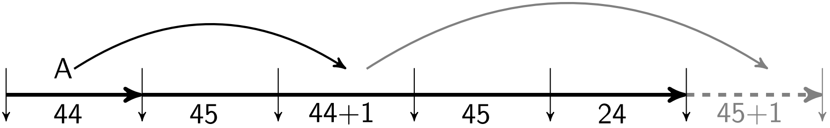

Legal

This can be iterated indefinitely: non-locality

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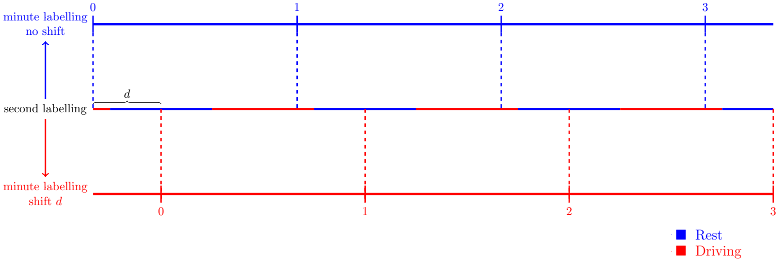


(51) Given a calendar minute, if DRIVING is registered as the activity of both the immediately preceding and the immediately succeeding minute, the whole minute shall be regarded as DRIVING.

(52) Given a calendar minute that is not regarded as DRIVING according to requirement 051, the whole minute shall be regarded to be of the same type of activity as the longest continuous activity within the minute (or the latest of the equally long activities).

Regulation (EU) 2016/799

Candidate: monadic second order logic MSO

Starting point

Borges, Conejero, Fernández-Duque, González, Joosten.
To drive or not to drive: A logical and computational analysis of European transport regulations. Information and Computation 280, 2021.

- naturally formalizes Regulation 561.
- model-checking in time $f(|\varphi|) \cdot |w|$, Parameterized Complexity
  where $f : \mathbb{N} \to \mathbb{N}$ is some computable function.
- but f grows very fast:

  Theorem *(Frick, Grohe 04)*
  Assume $P \neq NP$. Then $MC(\Sigma^*, MSO)$ is not decidable in time

$$f(|\varphi|) \cdot |w|^{O(1)}$$

  for elementary $f : \mathbb{N} \to \mathbb{N}$.

Hence MSO is not sufficiently tractable.

Candidate: linear time temporal logic LTL

Model-checking in time $O(|\varphi| \cdot |w|)$, but not sufficiently expressive and not sufficiently succinct

Example `Article 6.2: The weekly driving time shall not exceed 56 hours`

Straightforwardly formalized over words of length $1w$: disjunction of

$$\bigwedge_{d \leq D} \left(\bigwedge_{r_d \leq i < \ell_{d+1}} \bigcirc^i \neg d \wedge \bigwedge_{\ell_d \leq i < r_d} \bigcirc^i d \right)$$

for all $D \leq 1w$ and

all $r_0 := 0 \leq \ell_1 < r_1 < \cdots < \ell_D < r_D < \ell_{D+1} := 1w$ with

$$\sum_{1 \leq j \leq D} (r_j - \ell_j) \leq 56h$$

This has $> \binom{7 \cdot 24 \cdot 60}{56 \cdot 60} > 10^{2784}$ many disjuncts.

Warning

Algorithmic laws could use large constants for time constraints. Model-checking complexity should scale well with them.

FrameTitle

Using bisimulation techniques one can prove:

Theorem

There is no $\mathcal{L}_{\circ,\Box}$ formula equivalent to $\psi_{8.6}$ over the class of eventually resting models.

Theorem

All $\mathcal{L}_{\circ\mathsf{U}}$ formulas equivalent to $\psi_{8.6}$ have U-depth *at least* 1140.

The central computational problem of algorithmic law

Need to formalize activity sequences and laws

- formalize activity sequences are words $w \in \Sigma^*$ over a finite alphabet Σ
  e.g, dddrrw formalizes 6 minutes of activities in $\Sigma = \{d, r, w\}$.
- formalize a law by a sentence in a suitable logic L.

Need algorithm that decides the computational problem

$MC(\Sigma^*, L)$
Input: a word $w \in \Sigma^*$ and a sentence $\varphi \in L$
Problem: is w legal according to φ, i.e. $w \models \varphi$?

$\mathrm{MC}(\Sigma^*, L)$ is a formal model for algorithmic law (on activity sequences).
Question For which L is it good?

Which MC(Σ^*, L) are good models for algorithmic law?

Tractability

sufficiently fast model-checkers

fine-grained complexity analysis: parameterized complexity theory

important parameter: large time constants in law

Expressivity

test case: formalize Regulation 561

Naturality

readable sentences

sufficiently succinct

Stopwatch automata SWA: syntax

Stopwatch automaton $\mathbb{A}$

Q finite set of states including start, accept

X finite set of stopwatches

λ maps $q \in Q$ to $\lambda(q) \in \Sigma$

β maps $x \in X$ to bound $\beta(x) \in \mathbb{N}$

ζ is the set of $(x, q) \in X \times Q$ such that x is active in q

Δ is the set of transitions (q, g, α, q')

where $q, q' \in Q$, g is a guard, α is an action.

Assignment ξ maps $x \in X$ to $\xi(x) \leq \beta(x)$

Guard g is a set of assignments

Action α maps assignments to assignments

Stopwatch automata SWA: semantics

Transition system of $\mathbb{A}$

configurations (q, ξ)

switch edges $(q, \xi) \xrightarrow{0} (q', \xi')$

whenever $(q, g, \alpha, q') \in \Delta$, $\xi \in g$, $\xi' = \alpha(\xi)$

stay edges $(q, \xi) \xrightarrow{t} (q, \xi')$

where ξ' *increases* $\xi(x)$ *for* x *active in* q *to* $\min\{\xi(x)+t, \beta(x)\}$

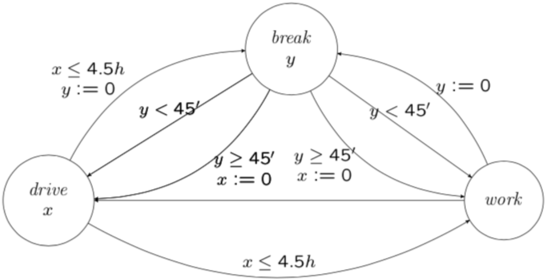
Computation $(q_0, \xi_0) \xrightarrow{t_0} (q_1, \xi_1) \xrightarrow{t_1} (q_2, \xi_2) \xrightarrow{t_2} \cdots \xrightarrow{t_{\ell-1}} (q_\ell, \xi_\ell)$

reads $w := \lambda(q_0)^{t_0}\ \lambda(q_1)^{t_1} \cdots \lambda(q_{\ell-1})^{t_{\ell-1}}$

accepts if $q_0 = \mathit{start}$, $\xi_0 \equiv 0$, $q_\ell = \mathit{accept}$, $q_i \neq \mathit{accept}$ for $i < \ell$.

Example: continuous driving

`Article 7 (1st part): After a driving period of four and a half hours a driver shall take an uninterrupted break of not less than 45 minutes,...`



$(drive, 00) \xrightarrow{3} (drive, 30) \xrightarrow{0} (break, 30) \xrightarrow{2} (break, 32) \xrightarrow{0} (work, 32) \xrightarrow{0}$
$(break, 30)$
reads *dddrr*

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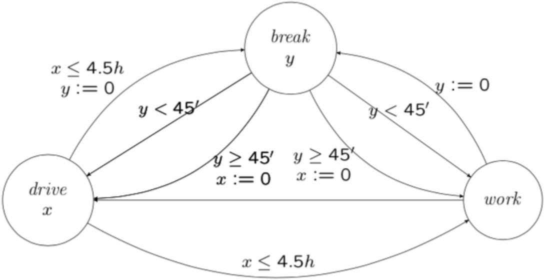

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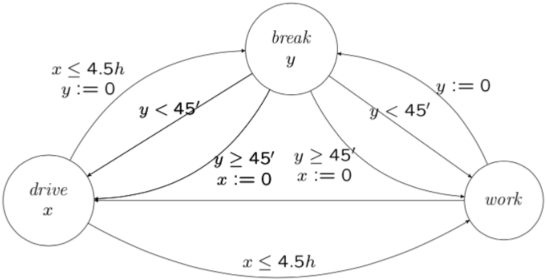

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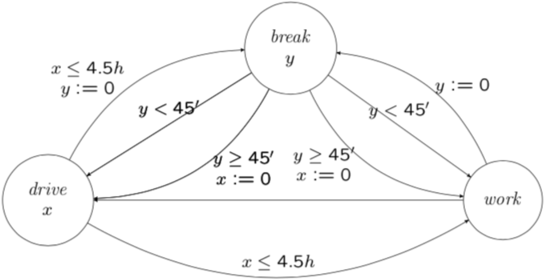

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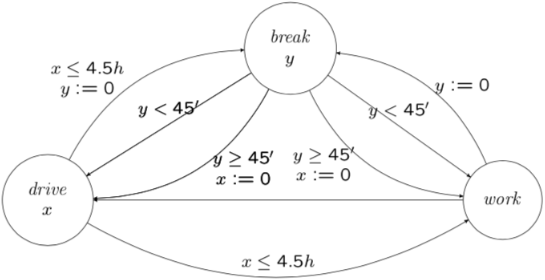

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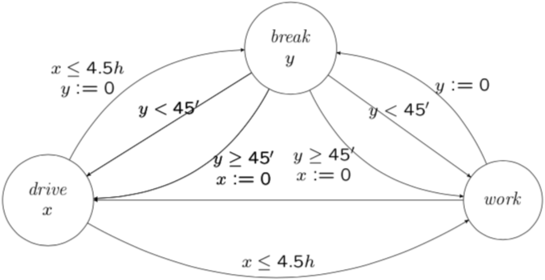

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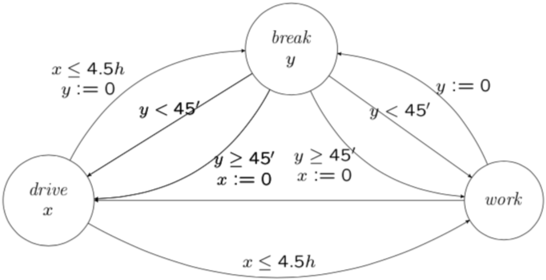

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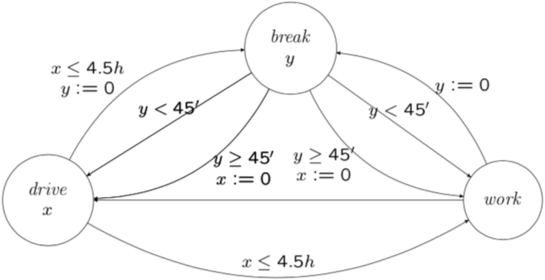

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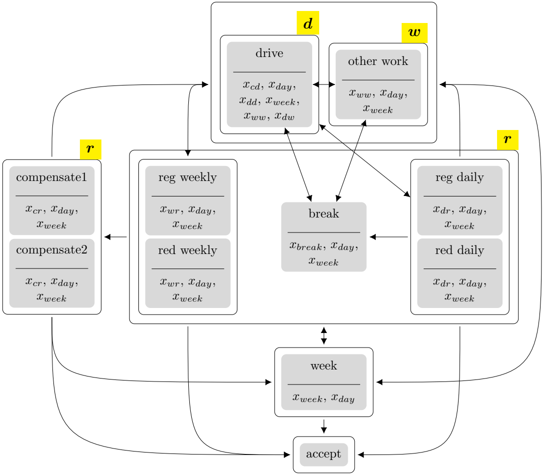
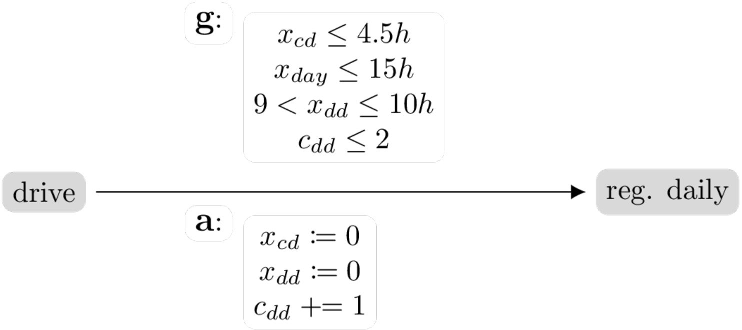
Automaton that accepts exactly the legal words according to Reg. 561



12 states
$>$ 100 transitions
34 stopwatches
23 are nowhere active:
bits
counters
registers

> 100 transitions



Expressivity and model-checking

Theorem

A set of words is accepted by an SWA iff it is definable in MSO.

Theorem

There is an algorithm that decides

> *Input: stopwatch automaton* $\mathbb{A}$ *and a word* w *over* Σ
>
> *Problem: does* $\mathbb{A}$ *accepts* w *?*

in time

$$O\big(|\mathbb{A}|^2 \cdot t^x \cdot |w|\big)$$

where

> $t :=$ *largest stopwatch bound of* $\mathbb{A}$
>
> $x :=$ *number of stopwatches of* $\mathbb{A}$

Consistency-checking

Theorem

There is an algorithm that decides

Input: SWAs $\mathbb{A}, \mathbb{B}$

Problem: is there a word accepted by both $\mathbb{A}$ *and* $\mathbb{B}$ *?*

in time

$O(|\mathbb{A}|^3 \cdot |\mathbb{B}|^3 \cdot t^x \cdot s^y)$

where

$t :=$ *largest stopwatch bound of* $\mathbb{A}$

$x :=$ *number of stopwatches of* $\mathbb{A}$

$s :=$ *largest stopwatch bound of* $\mathbb{B}$

$y :=$ *number of stopwatches of* $\mathbb{B}$

Scheduling

Theorem

There is an algorithm that decides

Input: *SWA* $\mathbb{A}$, *letter* $a \in \Sigma$, *word* w *over* Σ, $n \in \mathbb{N}$

Problem: *compute length* n *word* v *over* Σ *such that*

$\mathbb{A}$ *accepts* wv

v *maximizes* $\#_a(v)$

in time

$$O\big(|\mathbb{A}|^2 \cdot t^x \cdot (|w| + n)\big)$$

where

$t :=$ *largest stopwatch bound of* $\mathbb{A}$

$x :=$ *number of stopwatches of* $\mathbb{A}$

Lower bound

Know: MC(Σ^*, SWA) decidable in time $O\left(|\mathbb{A}|^2 \cdot t^x \cdot |w|\right)$
Doubt: Is t^x tolerable? Can it be improved?
Interesting instances have large t and small x.
Question: replace t^x by $100^{100 \cdot x} \cdot t^{100}$?

Theorem
Assume FPT $\neq$ W[1]. Let $f : \mathbb{N} \to \mathbb{N}$ be a computable function.
Then MC(Σ^*, SWA) cannot be decided in time

$$\left(|\mathbb{A}| \cdot f(x) \cdot t \cdot |w|\right)^{O(1)}.$$

Question: Can we hardwire large constants in the data structure using Hybrid Modal Logic?

What is certification?

- Is it just a matter of trust? (combined with some sanity checks and experience)

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- Is it just a matter of trust? (combined with some sanity checks and experience)
- Certificate $\Longrightarrow$ something is certain
- Verify $\Longrightarrow$ something is veridical

The impossibility of unrestricted certification

Alan Turing

- A mathematical theorem:

The impossibility of unrestricted certification

Alan Turing

- A mathematical theorem:
- Unrestricted certification is impossible.

Restricted certification is possible

We call a program P a *universal certifier* (wrt its language) when P takes two inputs

1. another program Q in a language compatible with P and,
2. a specification S in a language compatible with P that describes the behaviour of the program Q;

and, given two inputs Q and S, the program P outputs:

- **“YES”** if the program Q does what is said by S and, it will ouput
- **“NO”** if the program Q does something different as that what is claimed by S.

Theorem

There does not exist a universal certifier.
This holds for any reasonable class of languages.

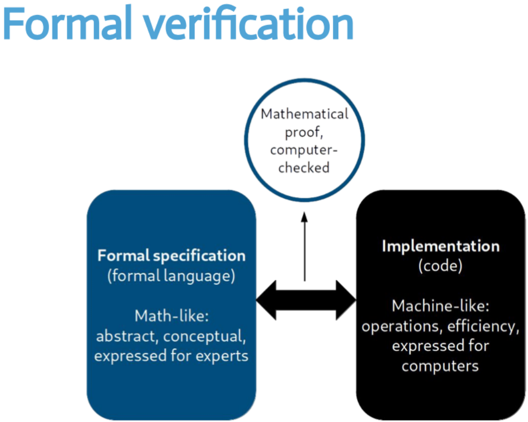
Formally verified software

Components of formally verified/certified software

- Σ A Specification: a non-ambiguous mathematical description of the input-output behaviour of the software
- Π Implementation: the code, the software, implementing the algorithm that does the work.
- Δ Proof: a mathematical proof that the program Π functions as claimed by Σ

The specification Σ is written in a formal language (in our case, the language of dependent types of the Coq proof assistant).
This begs the question: **How to make the specification more accessible to the general/judicial public?**

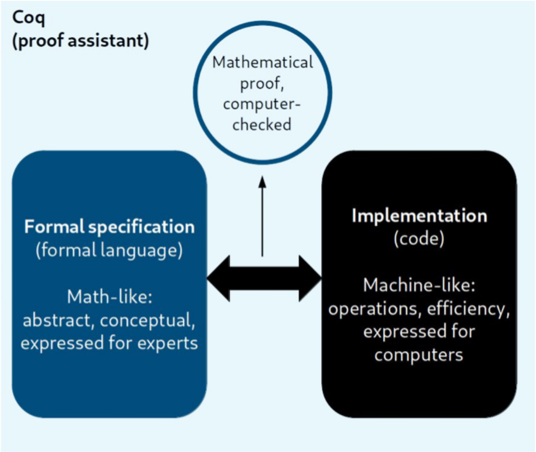
What is verification?



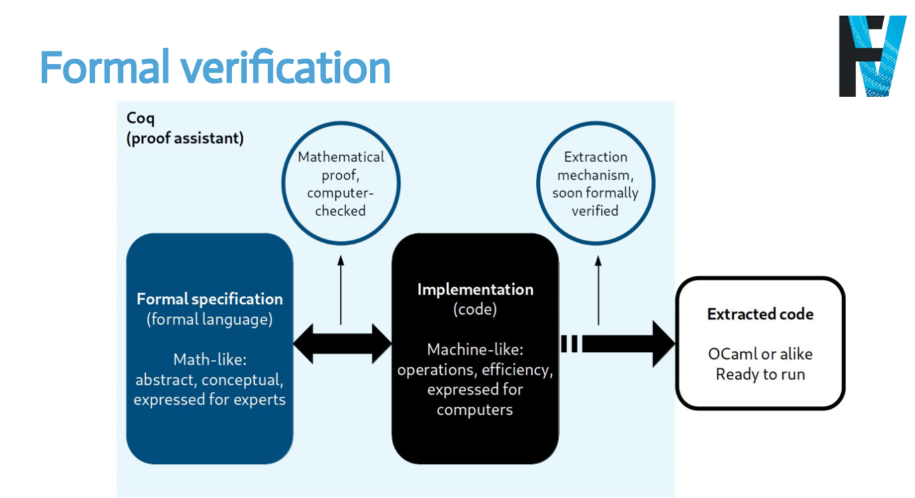
Slides FV: González Bedmar

What is verification?

Formal verification



What is verification?



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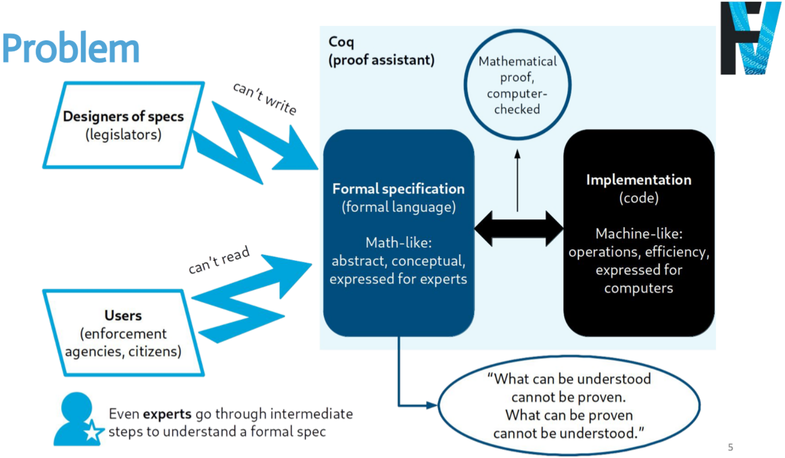
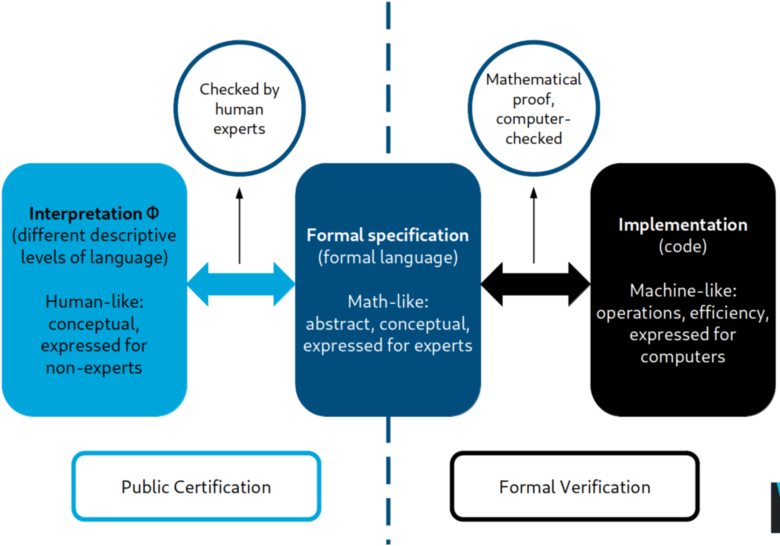
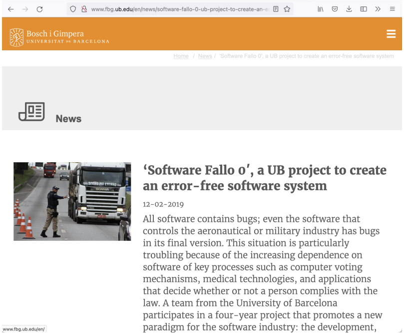
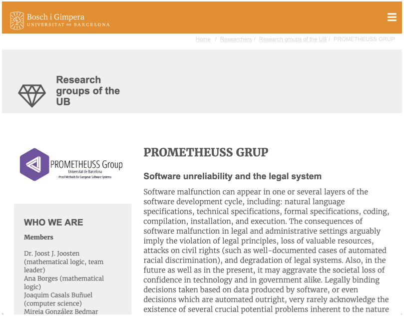


What is verification?



Ten years of research in Barcelona

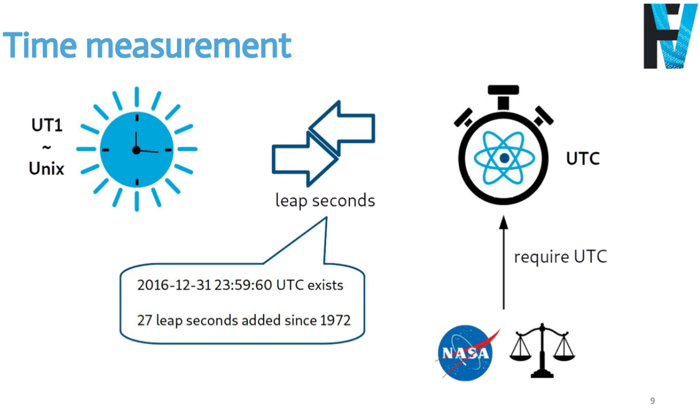




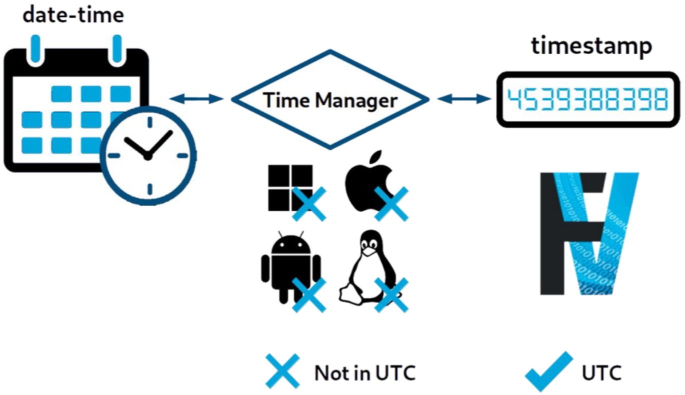
Covenant between the University of Barcelona (FBG), Formal Vindications S.L. & Guretruck S.L.

Closing conference

Time library

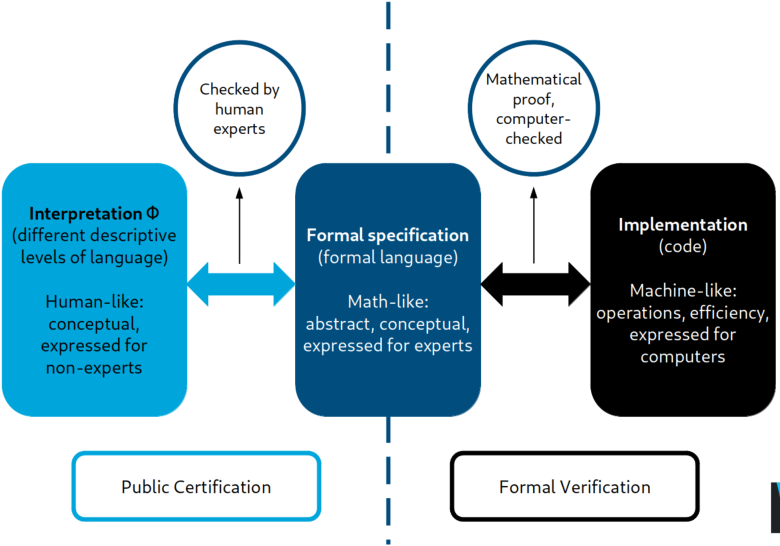


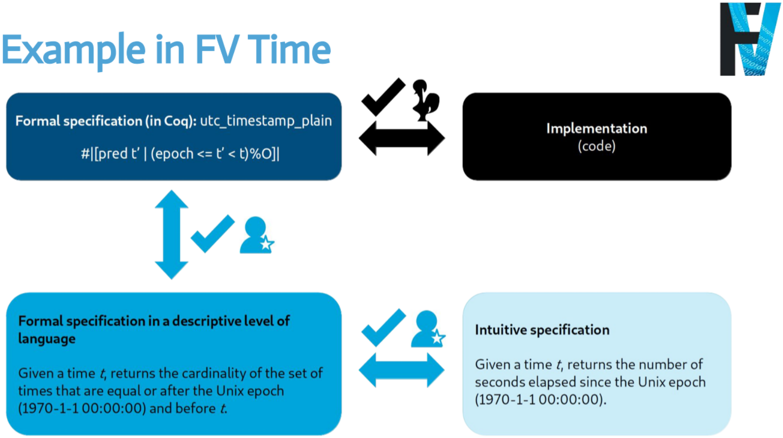
Time formats and managers



Most important feature: formally verified!

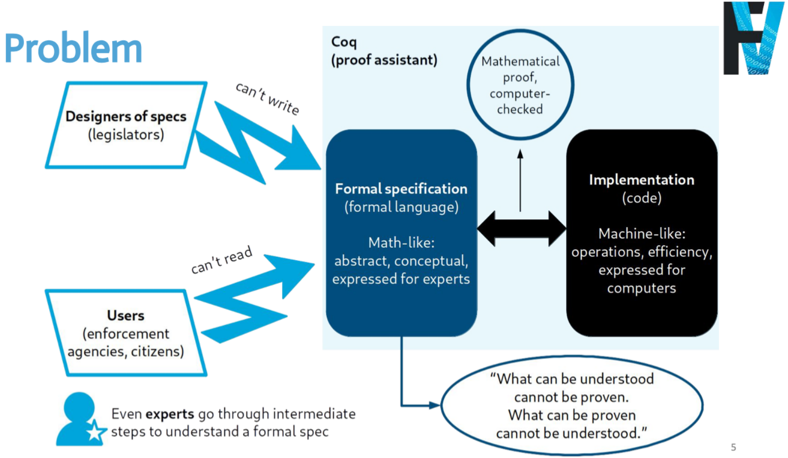
What is certification?



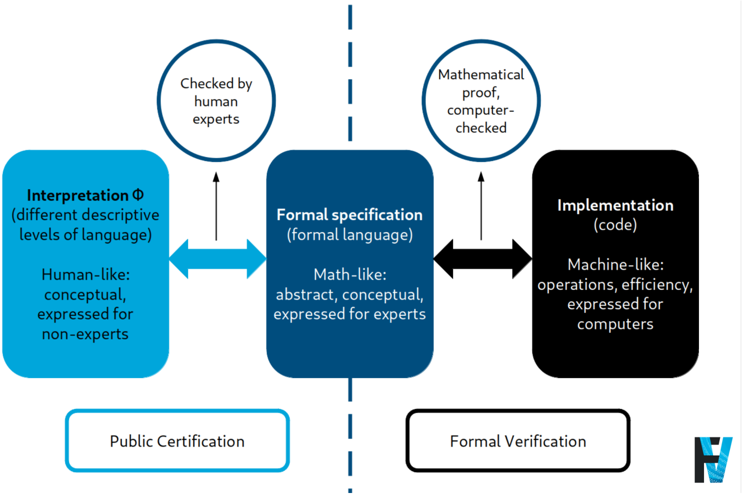


Around one-thousand times more expensive!

A central problem



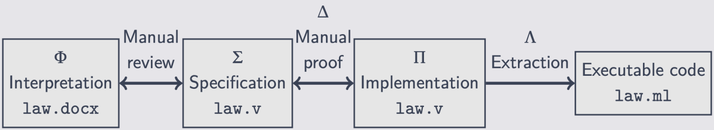
Public certification versus formal verification



Catala: A Shortcut For Legal Expert System Certification

The Usual Way to Produce Verified Software

Using Mireia Gonzáles Bedmar's conceptual framework from yesterday's presentation:



Catala's approach:

8

Slides Catala: Merigoux

➊ Catala: A Language Reviewable by lawyers

US Tax Code, Section 132, (c)(1) Qualified employee discount

The term "qualified employee discount" means any employee discount with respect to qualified property or services to the extent such discount does not exceed—
(A) in the case of property, the gross profit percentage of the price at which the property is being offered by the employer to customers

```
scope QualifiedEmployeeDiscount :
  definition qualified_employee_discount
    under condition is_property consequence equals
      if employee_discount >$ customer_price *$ gross_profit_percentage then
        customer_price *$ gross_profit_percentage
      else employee_discount
```

Can code be the law?

TENSION TABLE:

Computable laws:

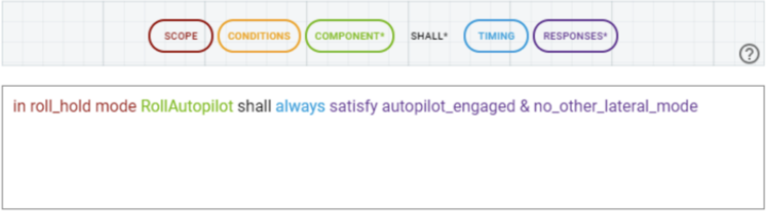
Language, software paradigm and legal principles

More accurate and exact but less undertandable for the general public

Specification Language	Programming paradigm	Legal Principles: Legal Certainty	Legal Principles: Accountability	Legal Principles: Contestability
Natural Language	Not Formally Verified	Decisions will probably not be consistent with the established legal framework. The text will be accessible and comprehensible to the public and authorities.	Automated decision won't be reliable and explainability will be difficult: the software is not comprehensible to the public, challenging the principle of transparency.	Right to contest turns almost impossible since authorities can't explain software decisions, which will be unreliable.
Technical Language	Not Formally Verified	Decisions will likely not be consistent with the established legal framework. The text will be less comprehensible to public and authorities.	Automated decision will be barely reliable and explainability will be difficult: the software is not comprehensible to the public, challenging the principle of transparency.	Right to contest turns almost impossible since authorities can't explain software decisions, which will be mostly unreliable.
Formal Language	Not Formally Verified	Decisions will probably be consistent with the established legal framework. The text will only be accessible to experts.	Automated decision will be quite reliable and explainability will be difficult: the software is not comprehensible to the public, challenging the principle of transparency.	Right to contest turns almost impossible since authorities can't explain software decisions, yet they will probably be working according to the law
Formal Language	Formally Verified	Decisions will be consistent with established legal framework. The text will only be accessible to experts	Automated decision will be reliable and explainability will be difficult, but it will be guaranteed that the software is the exact reproduction of its specification	Right to contest will be difficult since authorities can't explain software decisions, yet those are working according to the law

FRET: Formal Requirement Elicitation Tool

An attempt at bridging formal and natural language



FRET under the logic loupe: TIMING

TIMING is optional and specifies when response is expected

- immediately
- never
- eventually (the default reading when Timing is omitted)
- always
- within *n* time units
- for *n* time units
- after *n* time units
- ...

10 timing options

FRET under the logic loupe: SHALL

SHALL is mandatory

FRET under the logic loupe: COMPONENT

COMPONENT is mandatory

For example: The_Car

FRET under the logic loupe: CONDITION

CONDITION is an optional feature: a Boolean expression

For example: When Traffic_Light_Is_Red & No_Police_Car_Nearby

Three options: void condition, trigger condition, continual

FRET under the logic loupe: SCOPE

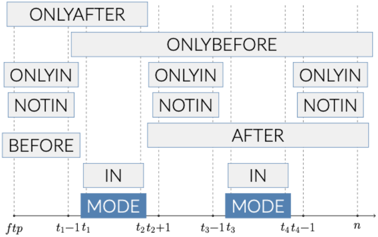
SCOPE is an optional feature: a finite collection of disjoint time intervals where the requirement is imposed

Built from a Boolean Mode M with an operator applied to it: $\mathcal{O}(M)$

- before M
- after M
- in M
- not in M
- only before M
- only after M
- only in M
- global (default scope when omitted)

8 operators in total

FRET under the logic loupe: SCOPE



FRET under the logic loupe: summary

Scope Conditions Component* Shall* Timing Responses*

Mandatory Fields		Optional Fields	
Component	system involved	**Timing**	when *response* should occur
Shall	obligation keyword	**Conditions**	applicability context: *void* (unconditional), *trigger* (activates on event), *continual* (holds while true)
Response	expected boolean behavior	**Scope**	temporal intervals of enforcement built from a *mode* (boolean) and an operator

FRET under the logic loupe: Semantic templates

8 scope operators
Condition or no condition
7 Timing options
give rise to

$$8 \times 3 \times 10 = 240$$

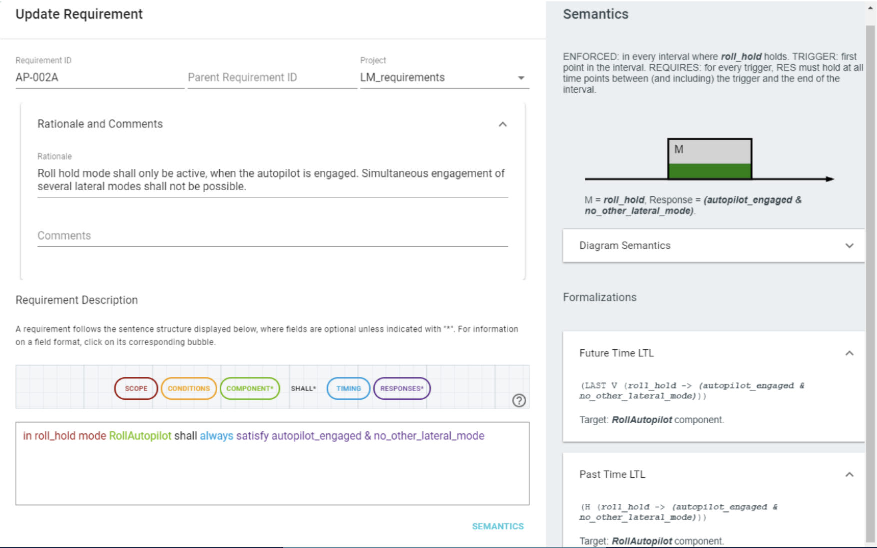
so-called semantic templates.
Each of the form

$$\mathsf{G}\Big(\mathcal{O}(M) \wedge \mathcal{C} \to \mathcal{T}(R)\Big)$$

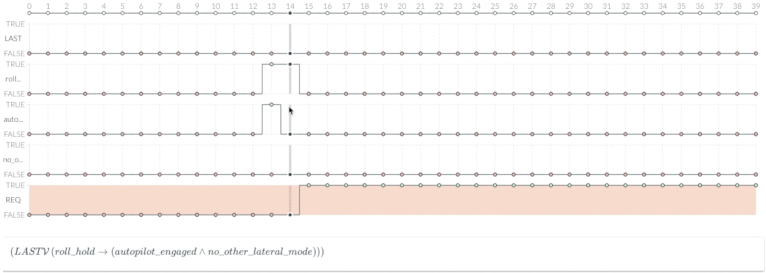
This is a simplified representation of the fragment of LTL covered by this version of FRET*

* (there are some small letters)

FRET interface: various language levels

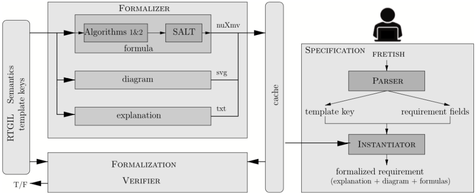


FRET: interactive sample testing



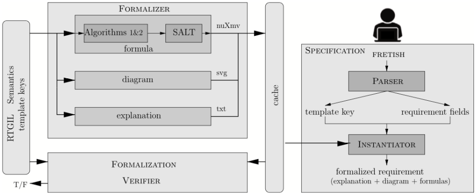
Users can test the behaviour of their FRET phrases

Serendipity has it...



- It seems that they wanted to tap into the RTGIL tool (Real Time Graphical Interval Logic)

Serendipity has it...



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- which I think communicates well with SALT (Structured Assertion Language for Temporal Logics)

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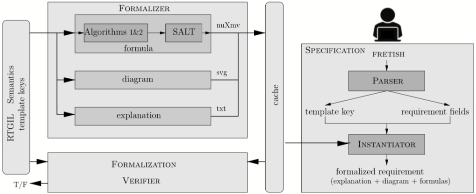

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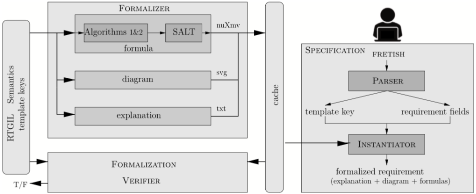

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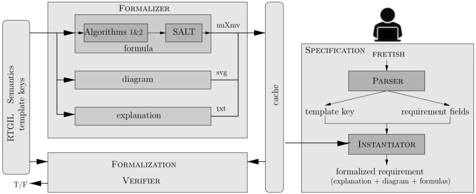

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- which I think communicates well with SALT (Structured Assertion Language for Temporal Logics)
- and nuXmv
- which is why they have curious ways of dealing with scopes (in my opinion)
- which make their way through the LTL translation it seems

A first translation algorithm

Table 1. (left) Scope endpoints. (right) pmLTL formulas associated with each endpoint. LAST+1 is not provided because our formulas do not use it.

Scope	LEFT	RIGHT
null	FTP	LAST+1
before	FTP	FFiM
after	FLiM	LAST+1
in	FiM	LiM
notin, onlyin	FNiM	LNiM
only before	FFiM	LAST+1
only after	FTP	FLiM

Symbol	Formula
FFiM	FiM and previous (historically (not M))
FLiM	LiM and previous (historically (not LiM))
FiM	M and (FTP or (previous not M))
LiM	not M and previous M
FNiM	not M and (FTP or previous M)
LNiM	M and previous (not M)
FTP	not previous true

```
generalform = (g-a) and (g-b)
g-a = historically (RIGHT implies previous BASEFORM_TO_LEFT)
g-b = ((not RIGHT) since inclusive required LEFT)
        implies BASEFORM_TO_LEFT
BASEFORM_TO_LEFT = (BASEFORM [since inclusive required LEFT]*)
```

Table 2. BASEFORMS without and with conditions. since_ir/since_er denote since inclusive/exclusive required, respectively.

Timing	BASEFORM	BASEFORM with conditions
immediately	LEFT implies RES	TRIGGER implies RES
always	[RES since_ir LEFT]*	NOCONDITION or (RES since_ir TRIGGER)
never	[always(not RES)]*	always(COND, (not RES))
eventually	[not ((not RES) since_ir LEFT)]*	[NOCONDITION or not ((not RES) since_ir TRIGGER)]*
for n	(once timed[$\leq n$] LEFT) implies RES	F_1 and F_2 $F_1 \equiv$ (((not LEFT) since_er TRIGGER) and (once timed[$\leq n$] TRIGGER)) implies RES $F_2 \equiv$ (COND and LEFT) implies RES
within n	((not RES) since_ir LEFT) implies (once timed[$<n$] LEFT)	(previous timed[$=n$] (TRIGGER and not RES)) implies (once timed[$<n$] (LEFT or RES))
after n	for(n, not (RES)) and within(n+1, RES)	for(COND, n, not (RES)) and within(COND, n+1, RES)

$$G\Big(\mathcal{O}(M) \wedge \mathcal{C} \to \mathcal{T}(R)\Big)$$

Second: slightly more promising

Table 2. Core formula definition for *null* condition or $cond \in \mathbb{B}$.

timing	$\Phi_{core}(timing, null, res, \phi_{left})$	$\Phi_{core}(timing, cond, res, \phi_{left})$
immediately	$\phi_{left} \rightarrow res$	$\phi_{trigger} \rightarrow res$
next	$(\mathcal{Y}\,\phi_{left}) \rightarrow res$	$(\mathcal{Y}\,\phi_{trigger}) \rightarrow (res \vee \phi_{left})$
always	res	$\phi_{noTriggers} \vee (res\,\mathcal{S}^{req}_{inc}\,\phi_{trigger})$
eventually	$\neg(\neg res\,\mathcal{S}^{req}_{inc}\,\phi_{left})$	$\phi_{noTriggers} \vee \neg(\neg res\,\mathcal{S}^{req}_{inc}\,\phi_{trigger})$
until(stop)	$(\neg stop\,\mathcal{S}^{req}_{inc}\,\phi_{left}) \rightarrow res$	$\phi_{noTriggers} \vee ((\neg stop\,\mathcal{S}^{req}_{inc}\,\phi_{trigger}) \rightarrow res)$
before(stop)	$stop \rightarrow ((\neg\phi_{left} \wedge \neg\,\mathcal{Y}\,(\neg res\,\mathcal{S}^{req}_{inc}\,\phi_{left})))$	$stop \rightarrow (\phi_{noTriggers} \vee (\neg\phi_{left} \wedge \neg\phi_{trigger} \wedge \neg\,\mathcal{Y}\,(\neg res\,\mathcal{S}^{req}_{inc}\,\phi_{trigger})))$
for(m)	$(O_{[0,m]}\,\phi_{left}) \rightarrow res$	$(O_{[0,m]}\,\phi_{trigger}) \rightarrow (\phi_{noTriggers} \vee res)$
within(m)	$(\neg res\,\mathcal{S}^{req}_{inc}\,\phi_{left}) \rightarrow (O_{[0,m-1]}\,\phi_{left})$	$\mathcal{Y}^m(\phi_{trigger} \wedge \neg res) \rightarrow (O_{[0,m-1]}(\phi_{left} \vee res))$

$$\mathsf{G}\Big(\mathcal{O}(M) \wedge \mathcal{C} \rightarrow \mathcal{T}(R)\Big)$$

Simplified architecture

Ex: MTL Formulas Translating FRETISH for Future Time on Infinite Traces

$$\mathcal{G}\left(\llbracket \mathsf{Mode} \rrbracket \wedge \mathsf{Cond} \rightarrow \mathcal{T}_\infty(\mathsf{Resp}, \llbracket \mathsf{Mode} \rrbracket)\right) \quad \text{if } \mathsf{Cond} \text{ is continual}$$

$$\mathcal{G}\left(\mathbf{ChangeTo}\left(\llbracket \mathsf{Mode} \rrbracket \wedge \mathsf{Cond}\right) \rightarrow \mathcal{T}_\infty(\mathsf{Resp}, \llbracket \mathsf{Mode} \rrbracket)\right) \quad \text{if } \mathsf{Cond} \text{ is trigger}$$

$\mathbf{ChangeTo}(\cdot)$	Activates when expression changes from false to true to capture the trigger dynamics
$\llbracket \mathsf{Mode} \rrbracket$	MTL translation of the Scope
$\mathcal{T}_\infty$	MTL translation of the Timing constraint

Simplified architecture: lookup tables

Ordinal	Scope operator	LTL definition
0	Global **Mode**	$\top$
1	In **Mode**	**Mode**
2	Not In **Mode**	$\neg$ **Mode**
3	Only In **Mode**	$\neg$ **Mode**
4	Before **Mode**	$\mathcal{H}\neg$ **Mode**
5	Only Before **Mode**	$\mathcal{O}$ **Mode**
6	After **Mode**	$\mathcal{O}(\neg\text{Mode} \wedge \mathcal{Y}\,\text{Mode})$
7	Only After **Mode**	$\mathcal{H}(\mathcal{Y}\,\text{Mode} \rightarrow \text{Mode})$

(a) Look-up table for FRETISH Scope

ID	Condition kind
0	void
1	continual
2	trigger

(b) Look-up table for FRETISH Condition

ID	FRETISH Timing	MTL formula
0	eventually	$[\![\text{Mode}]\!]\ \mathcal{U}\ \left(\text{Resp} \wedge [\![\text{Mode}]\!]\right)$
1	immediately	Resp
2	next	$\mathcal{X}\,\text{Resp} \vee \mathcal{X}\,\textbf{ChangeTo}\,\neg[\![\text{Mode}]\!]$
3	always	$(\mathcal{X}\,\textbf{ChangeTo}\,\neg[\![\text{Mode}]\!])\ \mathcal{R}\ \text{Resp}$
4	never	$(\mathcal{X}\,\textbf{ChangeTo}\,\neg[\![\text{Mode}]\!])\ \mathcal{R}\ \neg\text{Resp}$
5	within (d)	$\Diamond_{[0,d]}\text{Resp} \vee \left([\![\text{Mode}]\!]\ \mathcal{U}_{[0,d]}\neg[\![\text{Mode}]\!]\right)$
6	for (d)	$\Box_{[0,d]}\text{Resp} \vee \left((\text{Resp} \wedge [\![\text{Mode}]\!])\ \mathcal{U}_{[0,d]}\neg[\![\text{Mode}]\!]\right)$
7	after(d)	$\left(\Box_{[0,d]}\neg\text{Resp} \wedge \Diamond_{[d+1,d+1]}\text{Resp}\right) \vee \left((\neg\text{Resp} \wedge [\![\text{Mode}]\!])\ \mathcal{U}_{[0,d+1]}\neg[\![\text{Mode}]\!]\right)$
8	until (stopCond)	$\Box\text{Resp} \vee \left(\text{Resp}\ \mathcal{U}\left(\text{stopCond} \vee \textbf{StrictChangeTo}\left(\neg[\![\text{Mode}]\!]\right)\right)\right)$
9	before (stopCond)	$\left(\text{Resp}\ \mathcal{R}\ \neg\text{stopCond}\right) \vee \left(\left([\![\text{Mode}]\!] \wedge \mathcal{X}\neg[\![\text{Mode}]\!]\right)\ \mathcal{R}\ \neg\text{stopCond}\right)$

(c) Look-up table for FRETISH Timing for infinite trace when the Scope does not include "only".

Comparison and Occam

FRETISH: In Scope upon Condition Component shall before StopCondition satisfy Response

FRET's MTL: ((G ((! ((! Scope) & (X Scope))) | (X (((Scope & (X (! Scope))) V (((! Condition) & ((X Condition) & (! (Scope & (X (! Scope)))))) -> ((X (! (((! ((! StopCondition) & (Response | (Scope & (X (! Scope)))))) & (! (Scope & (X (! Scope))))) U StopCondition))) & (! (Scope & (X (! Scope)))))) & (Condition -> (! (((! ((! StopCondition) & (Response | (Scope & (X (! Scope)))))) & (! (Scope & (X (! Scope))))) U StopCondition))))))) & (Scope -> (((Scope & (X (! Scope))) V (((! Condition) & ((X Condition) & (! (Scope & (X (! Scope)))))) -> ((X (! (((! ((! StopCondition) & (Response | (Scope & (X (! Scope)))))) & (! (Scope & (X (! Scope))))) U StopCondition))) & (! (Scope & (X (! Scope))))))) & (Condition -> (! (((! ((! StopCondition) & (Response | (Scope & (X (! Scope)))))) & (! (Scope & (X (! Scope))))) U StopCondition))))))

FRET translation

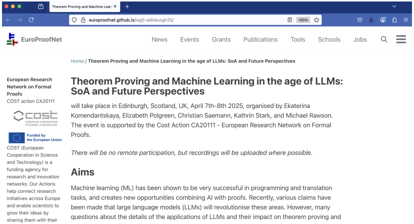
FRETISH: In Scope upon Condition Component shall before StopCondition satisfy Response
Simplified translation: G (ChangeTo (Scope & Condition) -> ((Response V ! StopCondition) | ((Scope & X ! Scope) V ! StopCondition)))

Our simplified translation (preparing Rocq implementation)

Spurious findings

- **Counter-intuitive constructs** whose informal interpretation diverges from their actual formal semantics, often leading to misunderstandings:
  - TheParcel shall within 1 day satisfy BeDelivered $\not\Rightarrow$ TheParcel shall eventually satisfy BeDelivered
  - **'Only In' Scopes defy logic**: Their semantics aren't logically grounded, but come from engineering intuition.
  - TheDriver shall after 3 hours of driving satisfy Rest forces that the rest cannot occur before 3 hours of driving.

Non-sharing incentives



16th century protagonists: Gerolamo Cardano, Niccolò Fontana (Tartaglia (stammerer)), Scipione del Ferro, Ludovico Ferrari, etc.

21st century protagonist: ChatGTP, DeepSeek, Mistral, Claude, . . .

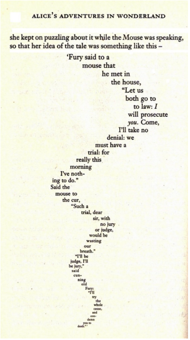
Belief revision appreciation revision

Succes from information theory: Self-attention with triple (queries, keys, values) in encoders and decoders

Controlled Natural Languages are promising to bridge reasoning and LLMs.

FRET simplification apology: The simpler they are, the better, the closer the formalisation to the CNL the better

Applied Logic in Law


ALICE'S ADVENTURES IN WONDERLAND

she kept on puzzling about it while the Mouse was speaking, so that her idea of the tale was something like this –

'Fury said to a
mouse that
he met in
the house,
"Let us
both go to
to law: *I*
will prosecute
you. Come,
I'll take no
denial: we
must have a
trial: for
really this
morning
I've noth-
ing to do."
Said the
mouse to
the cur,
"Such a
trial, dear
sir, with
no jury
or judge,
would be
wasting
our
breath."
"I'll be
judge, I'll
be jury,"
said
cun-
ning
old
Fury:
"I'll
try
the
whole
cause,
and
con-
demn
you to
death."'

It keeps the logician off the street

Fury said to the mouse: Civio vs Bosco

This article belongs to the debate » The Rule of Law versus the Rule of the Algorithm

02 April 2022

The Paradox of Efficiency: Frictions Between Law and Algorithms

On the 13th of January 2022, a Spanish Administrative court ruled in favour of algorithmic opacity. Fundación Civio, an independent foundation that monitors and accounts public authorities, reported that an algorithm used by the government was committing errors.[1)] BOSCO, the name of the application which contained the algorithm, was implemented by the Spanish public administration to more efficiently identify citizens eligible for grants to pay electricity bills. Meanwhile, Civio designed a web app to inform citizens whether they would be entitled for this grant.[2)] Thousands of citizens used this application and some of them reported that, while Civio's web app suggested

Ana Valdivia
Dr Ana Valdivia is a Postdoctoral Researcher at King's College London (ERC Security Flows). She examines how algorithms impact on people's life from a technical, political, and legal perspective.

Javier de la Cueva
Javier de la Cueva is a lawyer, lecturer and researcher in topics related to open knowledge, ethics and the digital world.

Explore posts related to this:
Algorithmic Efficiency, Algorithmic Justice, Rule of Law, Rule of the Algorithm

The Bosco computer program : errors in the computation of the social welfare bonuses

Least requirement: access to source code

In France it is mandatory to publish source code of software that is used in public administration.

However, access to source code will not resolve all problems

Thanks