



# UNIVERSITAT DE BARCELONA

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University of Barcelona

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The image shows the interior of a large, ornate cathedral. The nave is filled with rows of dark wooden pews with red upholstered seats. The floor is a light-colored checkered tile. The walls are covered in intricate murals and frescoes, including large scenes on the side walls and smaller figures in the upper sections. High arched windows with stained glass are visible along the top. At the far end, an altar area with red carpeting and a large arched opening is visible. A white rectangular sign with a blue border and the word 'WELCOME' in bold blue capital letters is centered in the foreground.

**WELCOME**



## THE REGULATIONS ARE LOCALLY STRICTLY STIPULATED BUT:

- ▶ Driving data is corrupted;
- ▶ Legal enforcement software flawed;
- ▶ Certain legal texts at the best ambiguous;
- ▶ And possibly not consistent or coherent.

**Could logic be of any help?**



# Mathematical logic can check laws with clear-cut ontologies for

$$\frac{\Gamma \vdash a : \langle G \rangle}{\Gamma \vdash P \triangleright \{c : G[p]\}} \quad \frac{k \in I \quad \Gamma \vdash e : S_k}{\Gamma \vdash c : \eta_k \triangleright \{c : T_k\}} \quad \frac{\Gamma \vdash a[p].P \triangleright \emptyset \quad \Gamma \vdash c : p! l_k(e). \eta_k \triangleright \{c : p! \{l_i(S_i).T_i\}_{i \in I}\}}{\Gamma \vdash c : p! l_k(e). \eta_k \triangleright \{c : p! \{l_i(S_i).T_i\}_{i \in I}\}} \quad [\text{T-ini/T-snd}]$$

► Consistency;

$$\frac{\Gamma \vdash c : p? \{l_i(x_i). \eta_i\}_{i \in I} \triangleright \{c : p? \{l_i(S_i).T_i\}_{i \in I}\}}{\Gamma \vdash c : p? \{l_i(x_i). \eta_i\}_{i \in I} \triangleright \{c : p? \{l_i(S_i).T_i\}_{i \in I}\}} \quad [\text{T-rcv}]$$

$$\frac{\Delta \text{ end-only}}{\Gamma \vdash c : 0 \triangleright \Delta} \quad \frac{\Gamma \vdash c : \eta \triangleright \{c : \text{end}\}}{\Gamma \vdash c : \underline{0}. \eta \triangleright \{c : \underline{\text{end}}.\text{end}\}} \quad [\text{T-0/T-yd}]$$

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 \end{array}$$

► Consistency;

► Tractability;

► Meeting physics constraints;

► Certificate that software meets the specification.

# OUTLINE OF MY TALK:





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## ■ Intro

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- Intro
- Bugs in Software

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- Intro
- Bugs in Software
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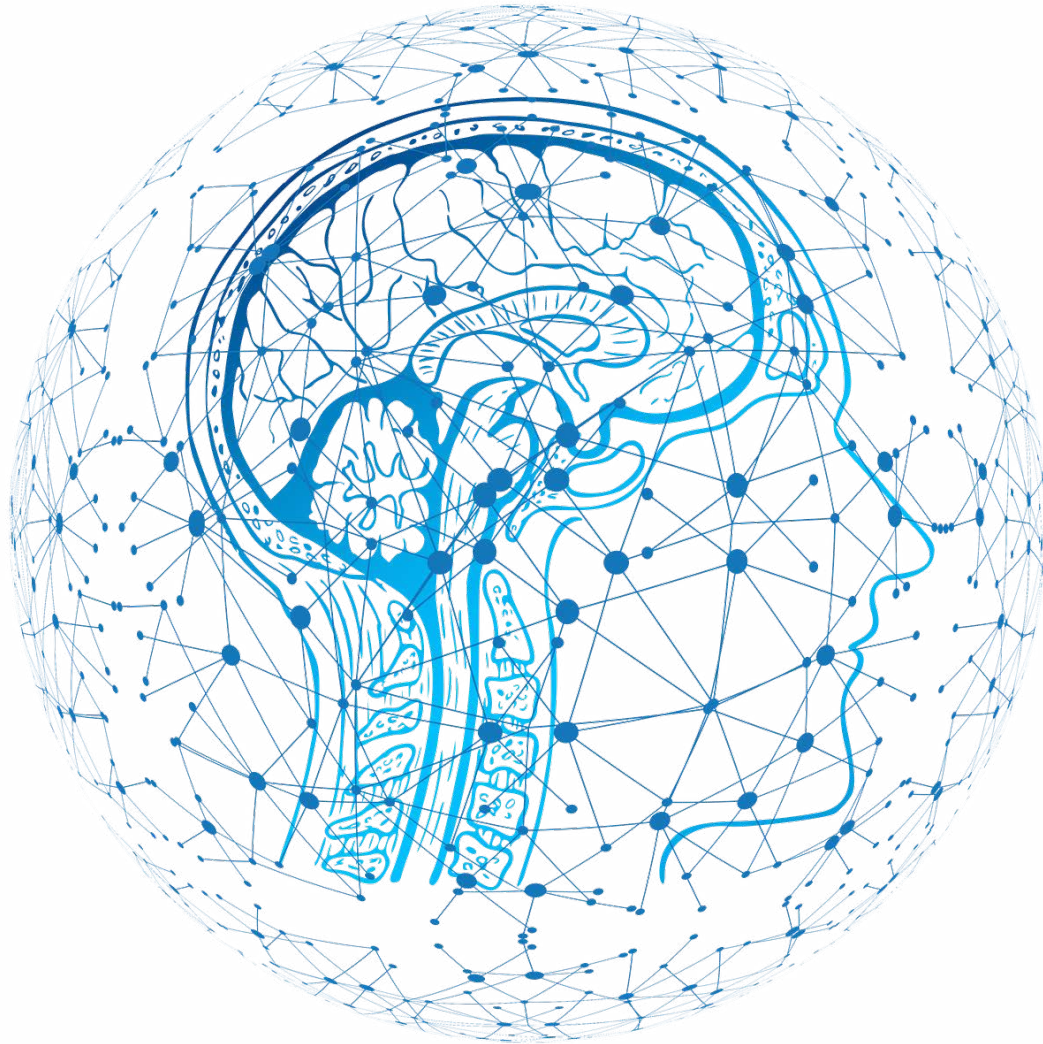


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- Intro
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Formally Verified Time Library

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- Intro
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- The future ahead of us

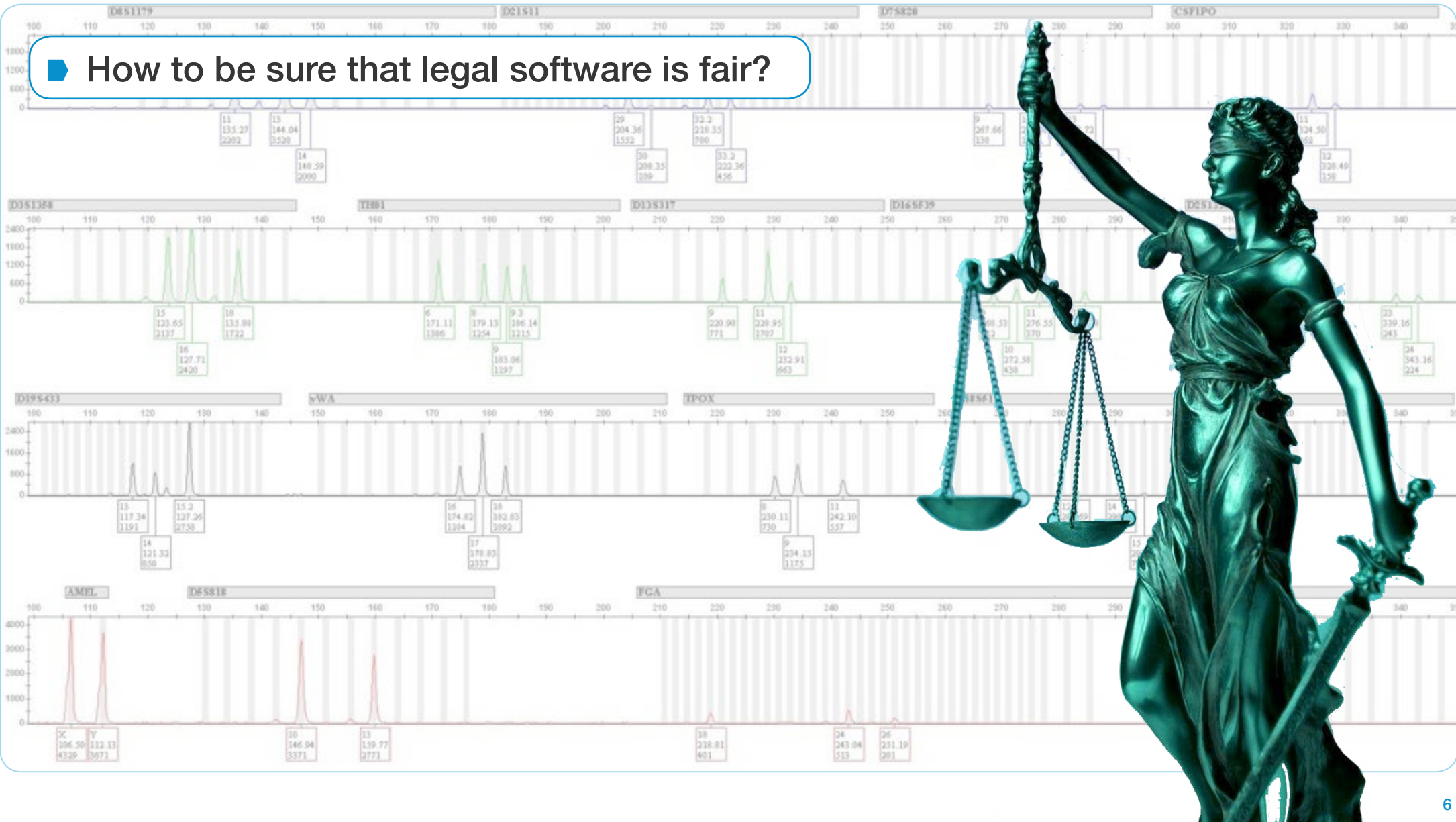




**Critical software should be error-free!**

# AI in society and law

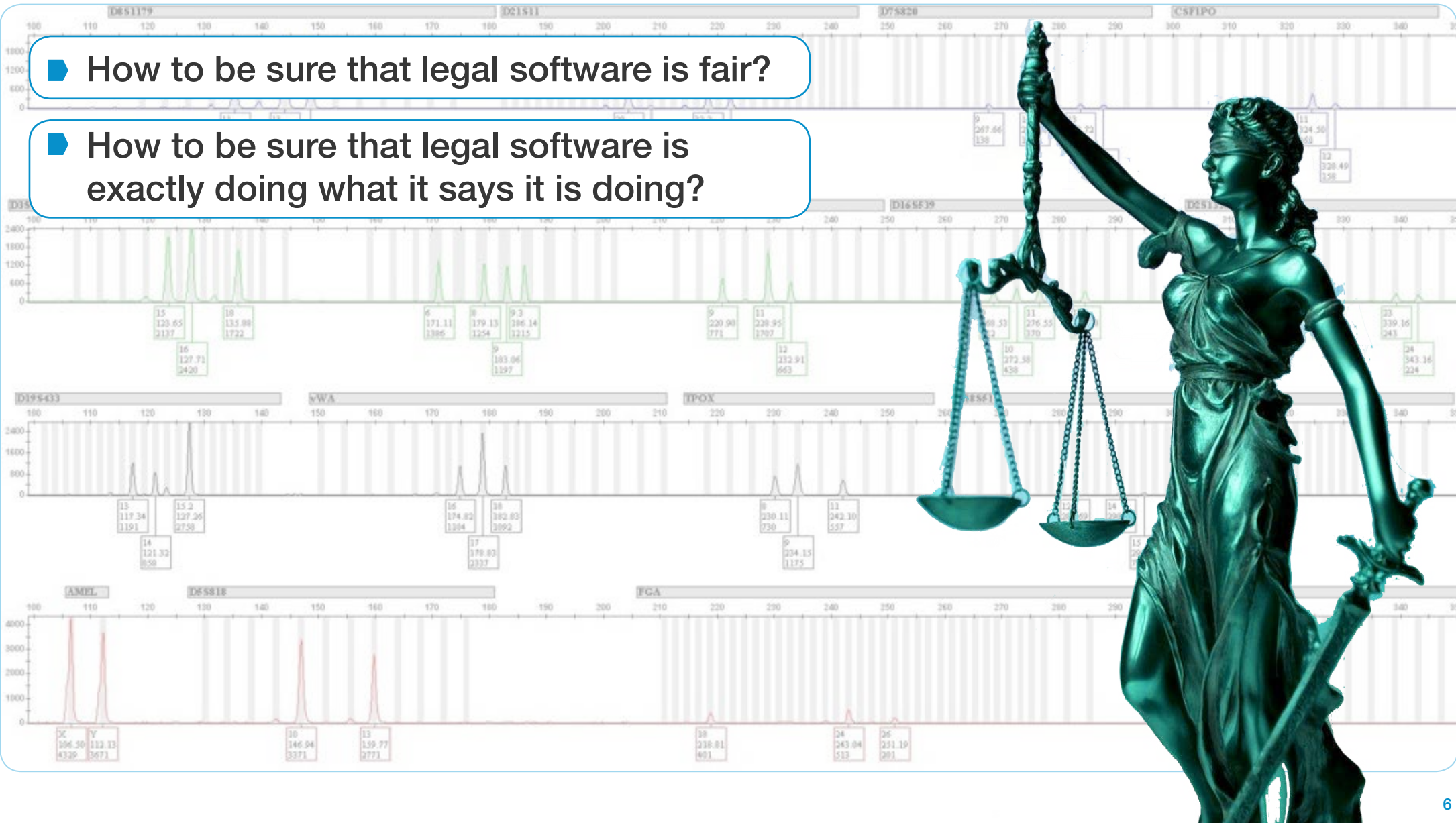
► How to be sure that legal software is fair?



# AI in society and law

► How to be sure that legal software is fair?

► How to be sure that legal software is exactly doing what it says it is doing?





# AI in society and law

- ▶ How to be sure that legal software is fair?
- ▶ How to be sure that legal software is exactly doing what it says it is doing?
- ▶ How to provide full legal transparency to legal software without necessarily disclosing the proprietary source code?





# IMPERATIVE PARADIGM.

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when computers where invented.**

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- Repeat process while some condition holds.



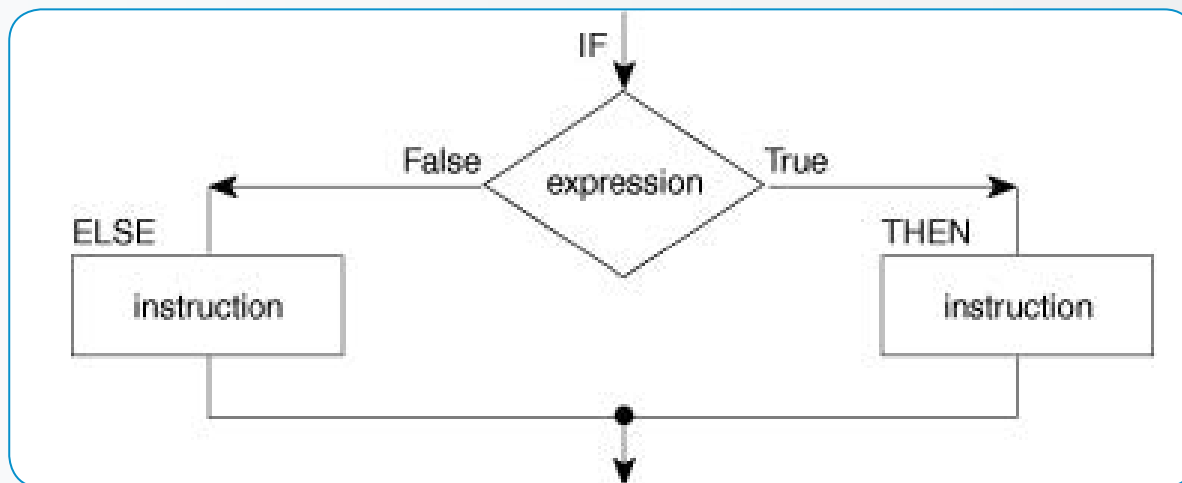


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- Store this number in the memory.
- Move this number to another location in the memory.
- Repeat process while some condition holds.
- Etc.



# FUNCTIONAL PARADIGM.

**Functional programming is a more advanced paradigm that is close to the mathematical language.**

**We do not tell the computer how to operate.**

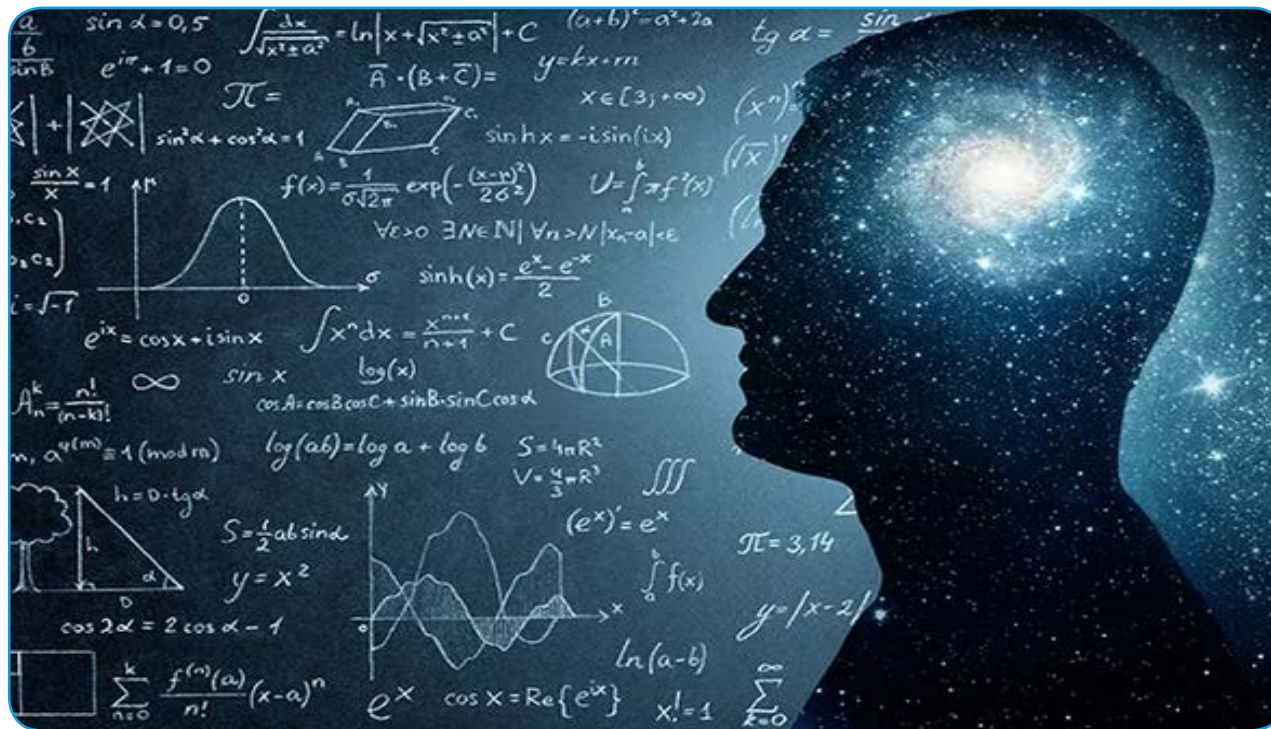
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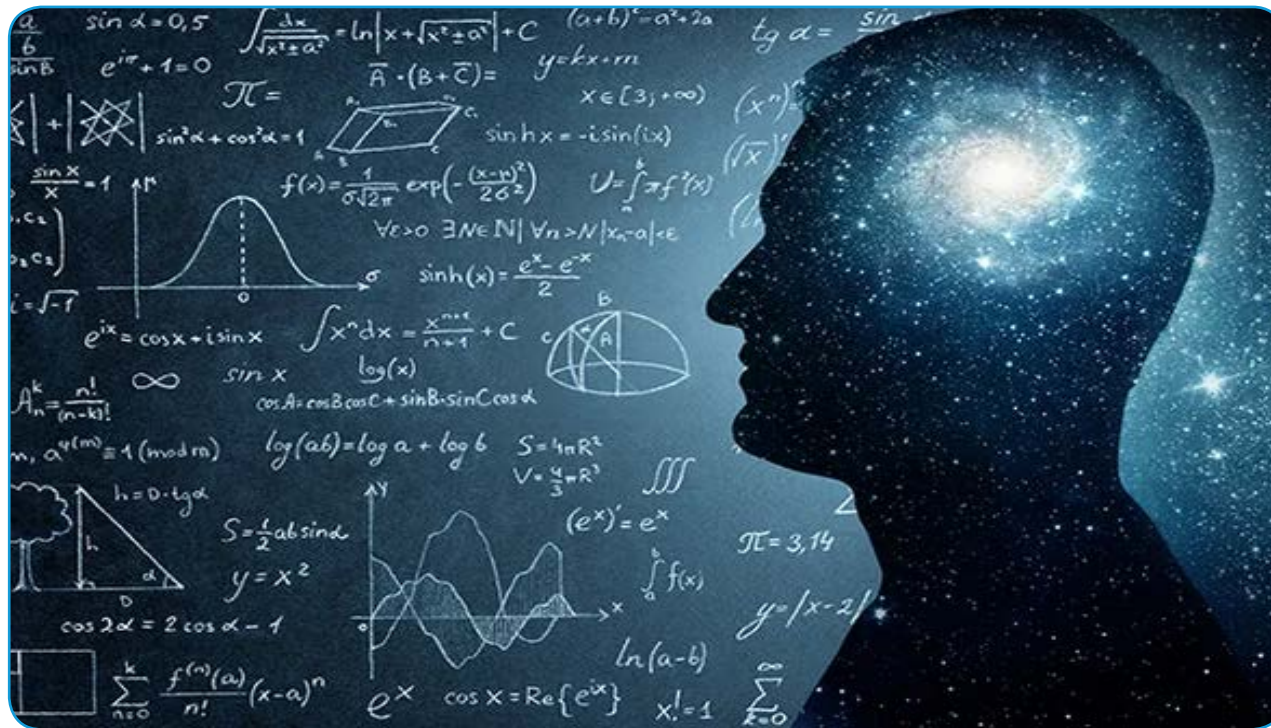
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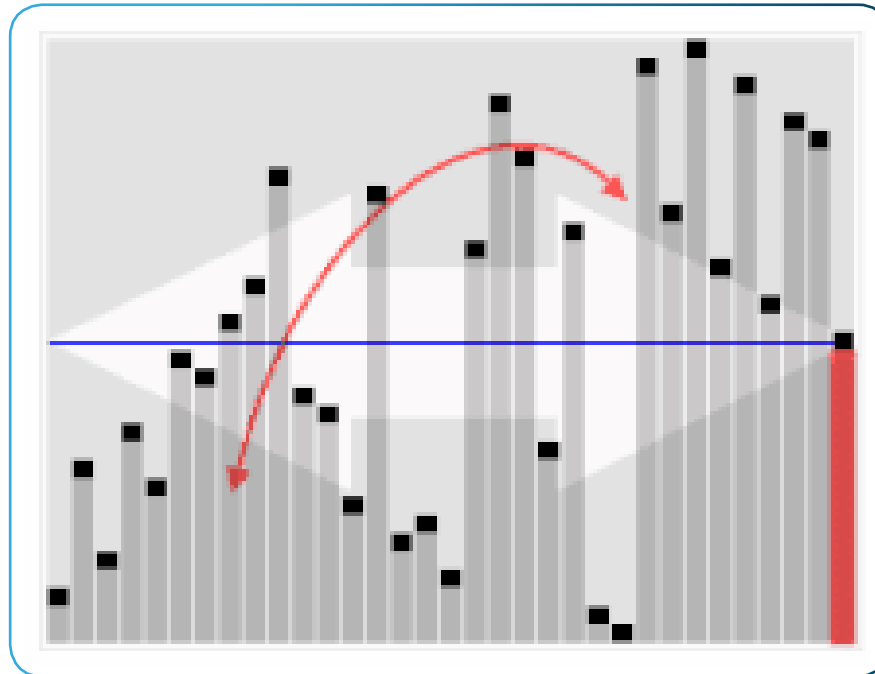


- **Advantage:** We can easily reason about our programs and prove their correctness.



# TEST CASE: SORTING ALGORITHM

- Quicksort is an efficient algorithm which sorts lists of numbers in lexicographical order.



FOR INSTANCE, APPLYING QUICKSORT TO [2,6,1,7] GIVES [1,2,6,7]

# IMPERATIVE VS FUNCTIONAL

## IMPERATIVE CODE:

```
algorithm quicksort(A, lo, hi) is  
  if lo < hi then  
    p := partition(A, lo, hi)  
    quicksort(A, lo, p)  
    quicksort(A, p + 1, hi)
```

```
algorithm partition(A, lo, hi) is  
  pivot := A[⌊(hi + lo) / 2⌋]  
  i := lo - 1  
  j := hi + 1  
  loop forever  
    do  
      i := i + 1  
      while A[i] < pivot  
      do  
        j := j - 1  
        while A[j] > pivot  
        if i ≥ j then  
          return j  
  swap A[i] with A[j]
```

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```

## FUNCTIONAL CODE:

```
algorithm quicksort (l : list) :=  
  case l of  
    [] -> []  
    [a] -> [a]  
    (a :: as) -> quicksort [x ∈ l : x ≤ a] ++  
                      quicksort [a ∈ l : x > a]
```

DEPENDENT TYPE THEORY CAN REASON ABOUT FUNCTIONAL CODE AND CAN BE USED TO PROVE CORRECTNESS OF THE CODE

# VERIFICATION OF FUNCTIONAL CODE

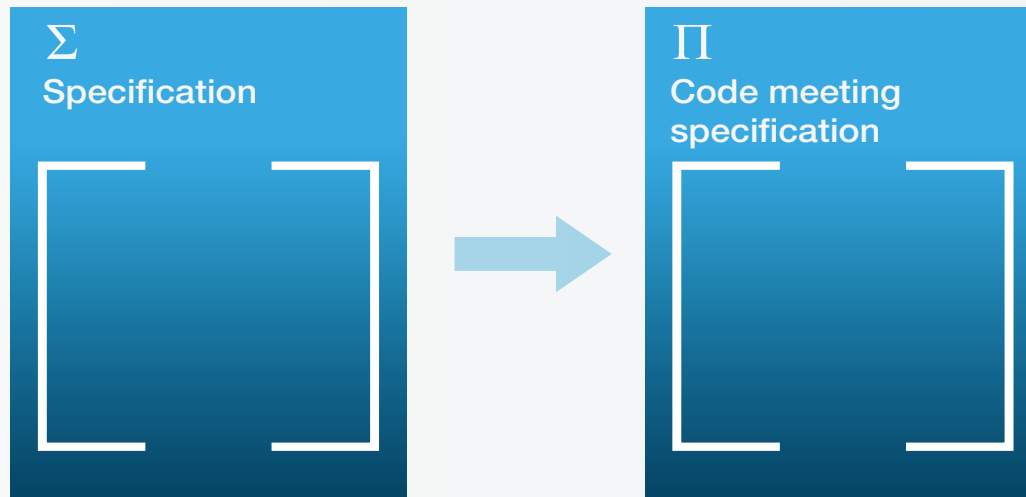
$\Sigma$

Specification

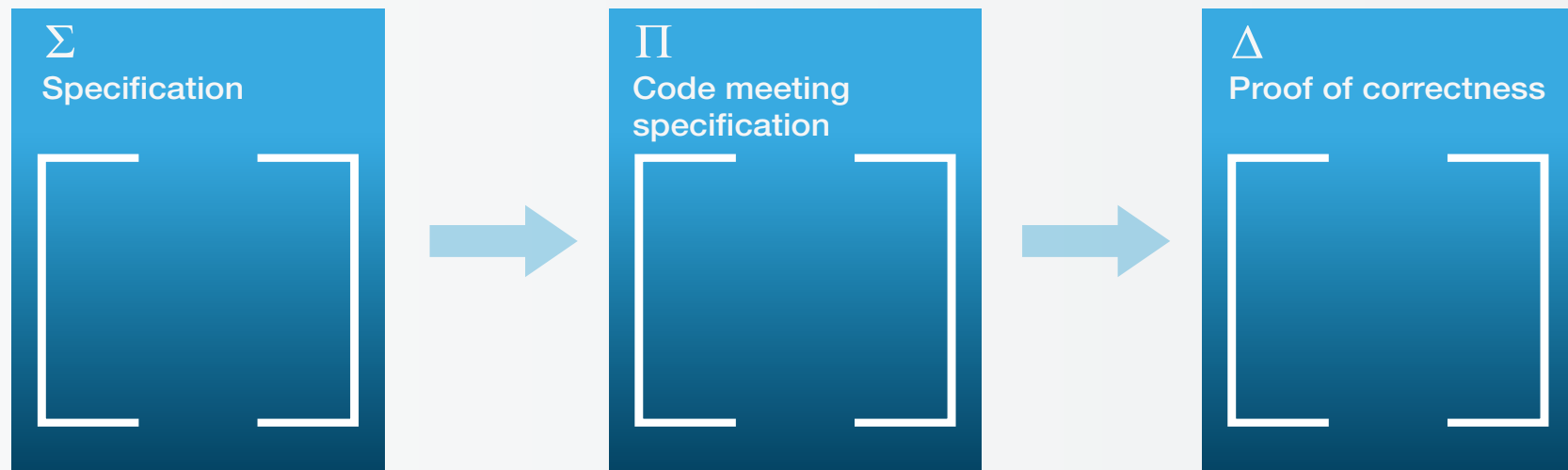




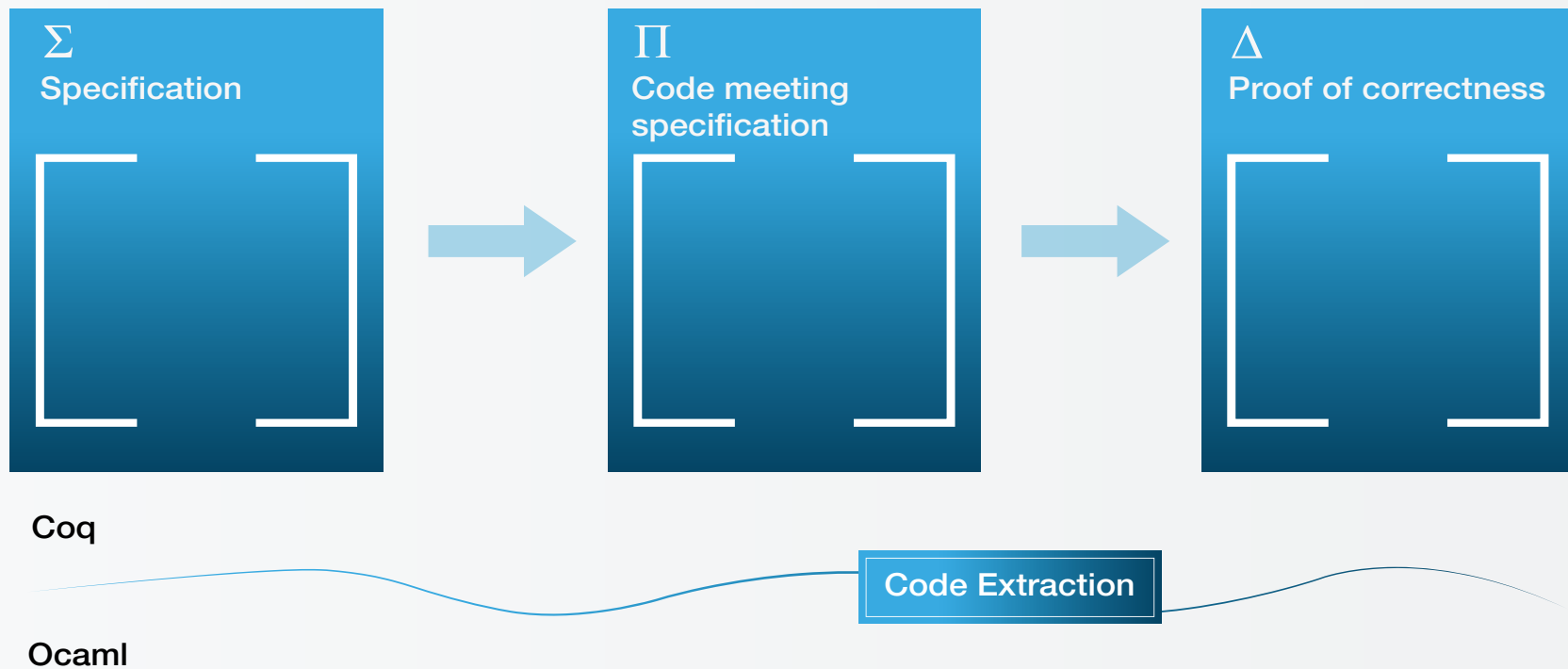
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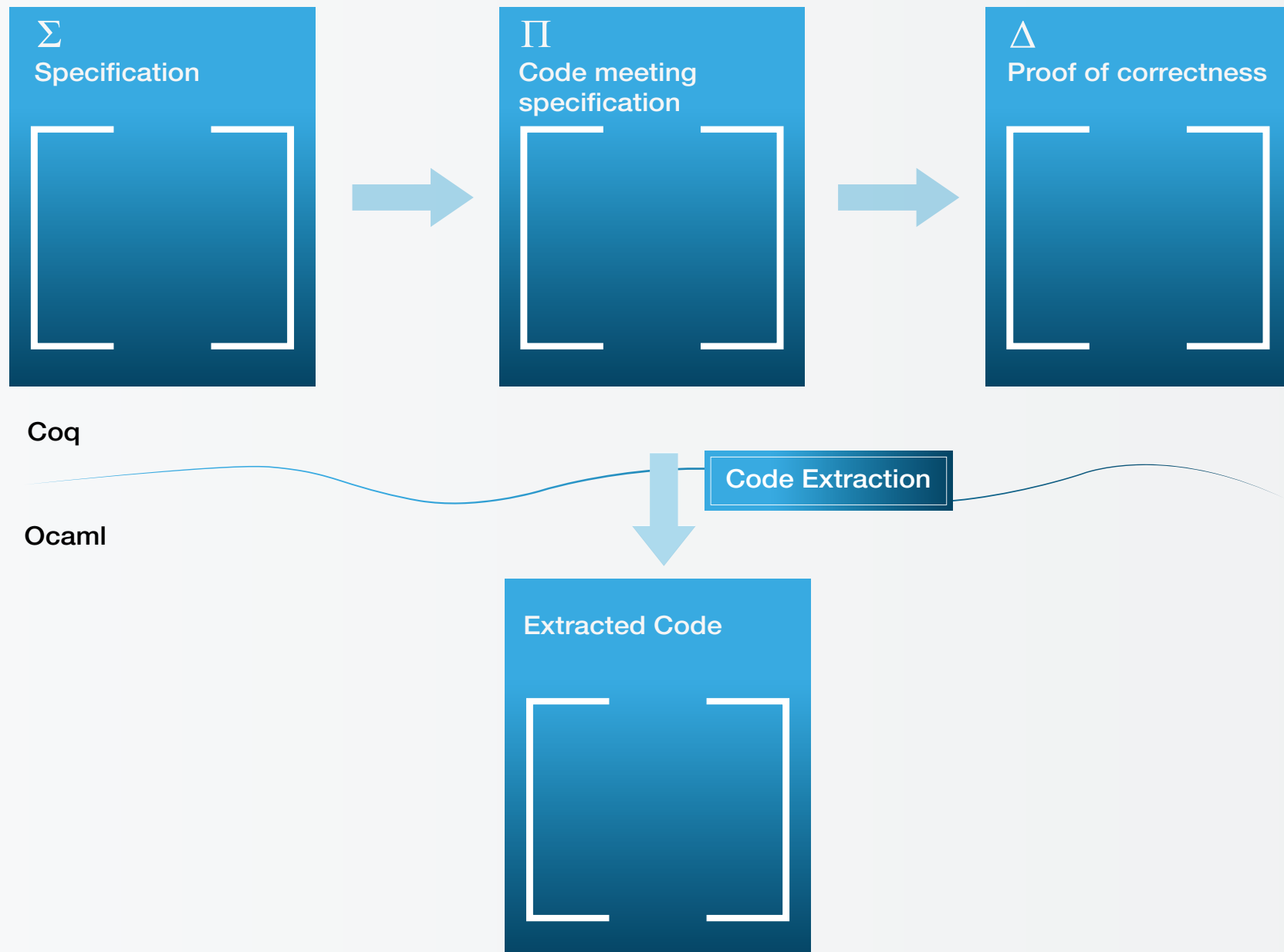
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# EXAMPLE THEOREMS

```
Lemma leq_trans a b c : a <= b -> b <= c -> a <= c.  
Proof. by elim: a b c => [|i IHn] [|b] [|c] //;  
      apply: IHn b c. Qed.
```



# EXAMPLE THEOREMS

**Lemma** leq\_trans a b c : a <= b -> b <= c -> a <= c.

**Proof.** by elim: a b c => [|i IHn] [|b] [|c] //;

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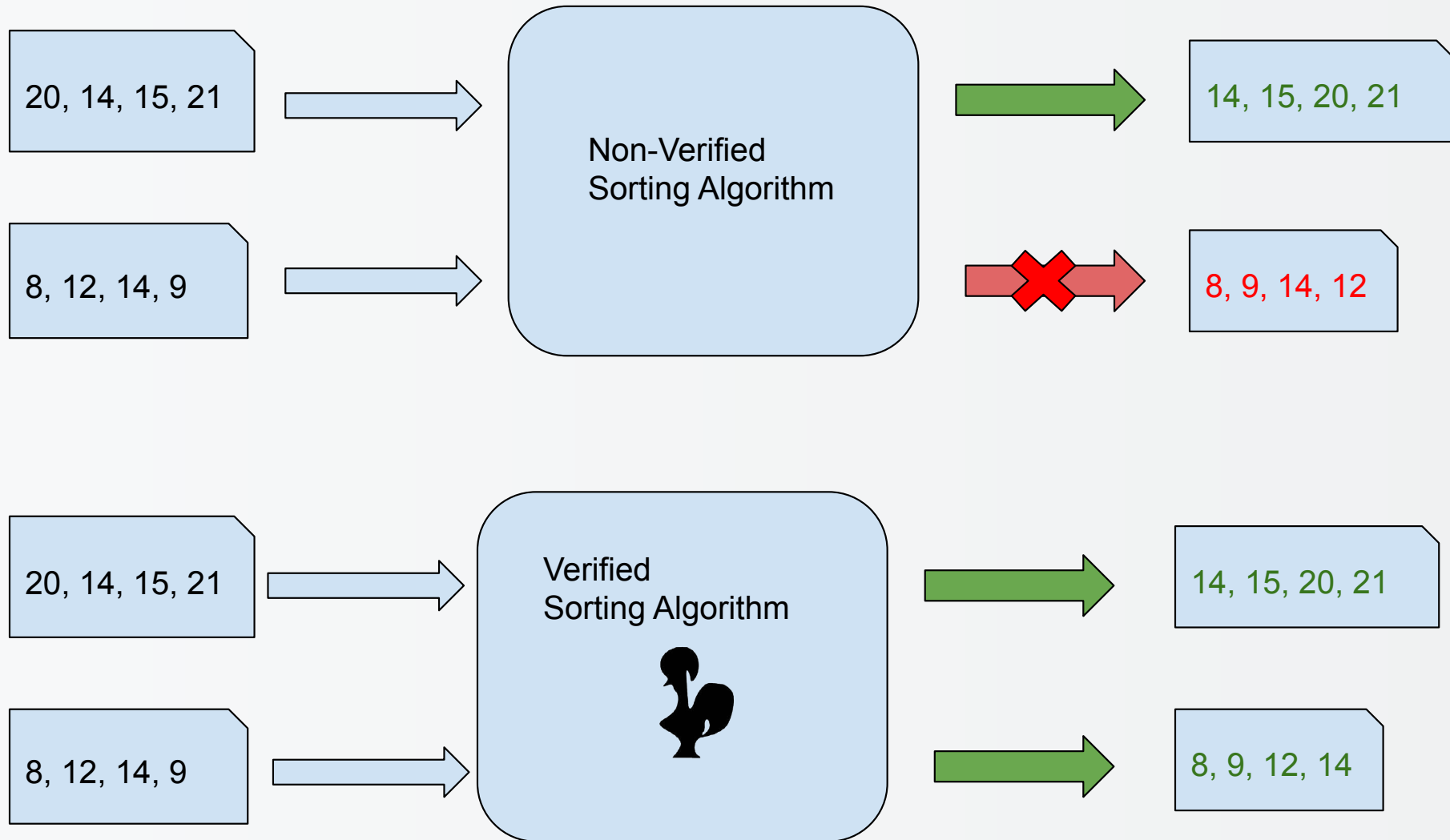
**Lemma** quicksort\_correct :

  forall l,

  permutation l (quicksort l)

  /\ is\_sorted (quicksort l).

# THE GOAL



# TIME MANAGEMENT IS IMPORTANT



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- Time management is a crucial part of many industry projects.
- We have developed our own verified library to be integrated in a bigger project.
- Bigger project: legal software involving time and durations  
for example  
full tachograph software.



# SPECIFICATION

| `Rec.WordF n`  $\Rightarrow$

- Mathematical definitions
- Blueprint of the running software
- Unambiguously defines software intended behaviour

| `Rec.ArrayF t' n0`  $\Rightarrow$

```
(fix array_of_word n from :=  
  match n with  
  | 0  $\Rightarrow$  Modify (InitSliceWithCapacity n0)  $\wedge$ (vdst)  
  | S n'  $\Rightarrow$   
    Declare (go_rec_type t') ( $\lambda$  vt'  $\Rightarrow$   
      (array_of_word n' (from + Rec.len t');  
       go_of_word t' vt' vsrc from;  
       Modify Append  $\wedge$ (vdst, vt'))))  
  end) n0 from
```

| `Rec.RecF fs`  $\Rightarrow$

```
(fix rec_of_word fs vdst from :=  
  match fs with  
  | []  $\Rightarrow$  Modify (@SetConst (Struct []) tt)  $\wedge$ (vdst)  
  | (_, f) :: fs'  $\Rightarrow$   
    Declare (go_rec_type f) ( $\lambda$  vf  $\Rightarrow$ 
```

# SPECIFICATION

```
| Rec.WordF n ⇒
```

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```
| Rec.ArrayF t' n0 ⇒
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- Given a time in format Year- Month- Day- hour : minute : second, obtain the number of seconds elapsed since 1970-01-01-00:00:00 until then

- Mathematical definition:

$$\text{timestamp}(T) := |\{t : 1970-01-01-00:00:00 \leq t < T\}|$$

```
      go_of_word v vt vdst from,
      Modify Append ^ (vdst, vt'))))
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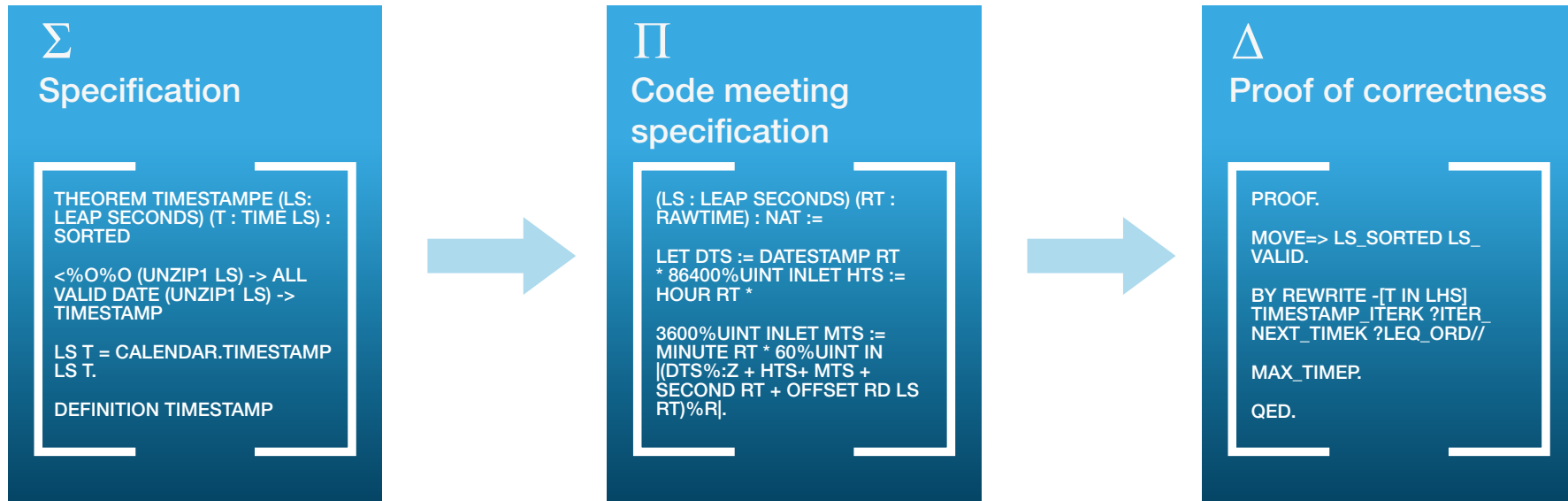
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go_of_word v vst from,  
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```

- Different implementations might correspond to the same specification
- Is not obvious when the implementation satisfies its specification

```
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| [] ⇒ Modify (@SetConst (Struct []) tt) ^ (vdst)  
| (_, f) :: fs' ⇒  
  Declare (go rec type f) (λ vf ⇒
```



```
Definition timestamp (ls : leap_seconds) (rt : rawTime) :  
nat :=  
  let Dts := datestamp rt * 86400%uint in  
  let hts := hour rt * 3600%uint in  
  let mts := minute rt * 60%uint in  
  |(Dts%:Z + hts + mts + second rt + offset_rd ls rt)%R|.
```



Coq

Ocaml

Code Extraction

## Extracted Code

```

LET TIMESTAMP RT =
LET DTS =
MULN (DATESTAMP (DATE_OF_
TIME RT)) (OF_UINT (D8 (D6 (D4
(D0 (D0
NIL)))))) IN
LET HTS = MULN RT.HOUR (OF_
UINT (D3 (D6 (D0 (D0 NIL)))))) IN

```

# TIME LIBRARY FUNCTIONALITY



**Time Library is formally verified with an ample functionality:**

- Conversion functions (4);
- Shift functions (6);
- Add functions (6);
- Auxiliary functions (174).



## Some statistics about the library and corresponding extraction:

- Estimated programming hours 30.000 = 41 formal months;
- Nr of lines of Coq code and proofs= 4114 (4821 incl. extraction directives);
- Nr of lemmata/theorems = 190 (278 incl. extraction directives);
- Nr of lines of extracted code = 1363 (down from an earlier ( $\pm$ ) 100 MB)

**A particularly hard function to deal with was `yoe_of_doe`**

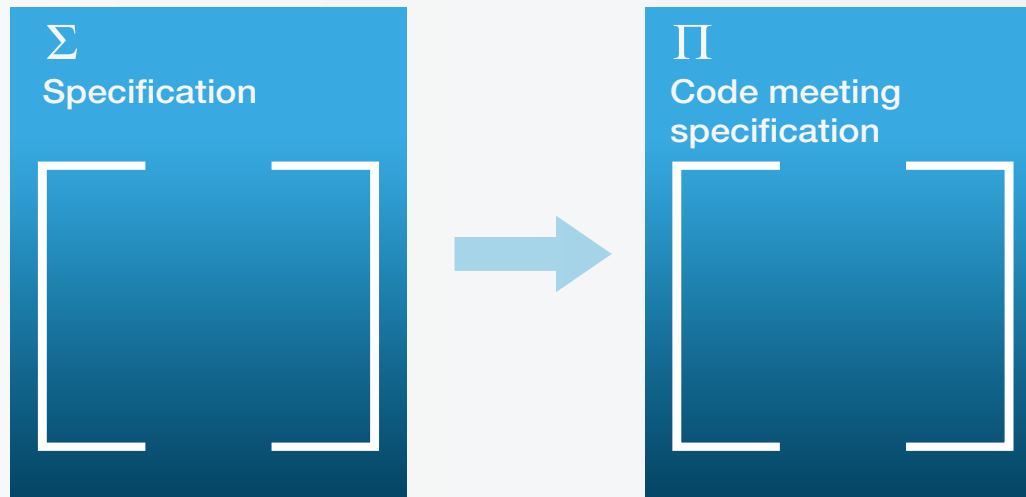
# VERIFICATION OF FUNCTIONAL CODE

$\Sigma$

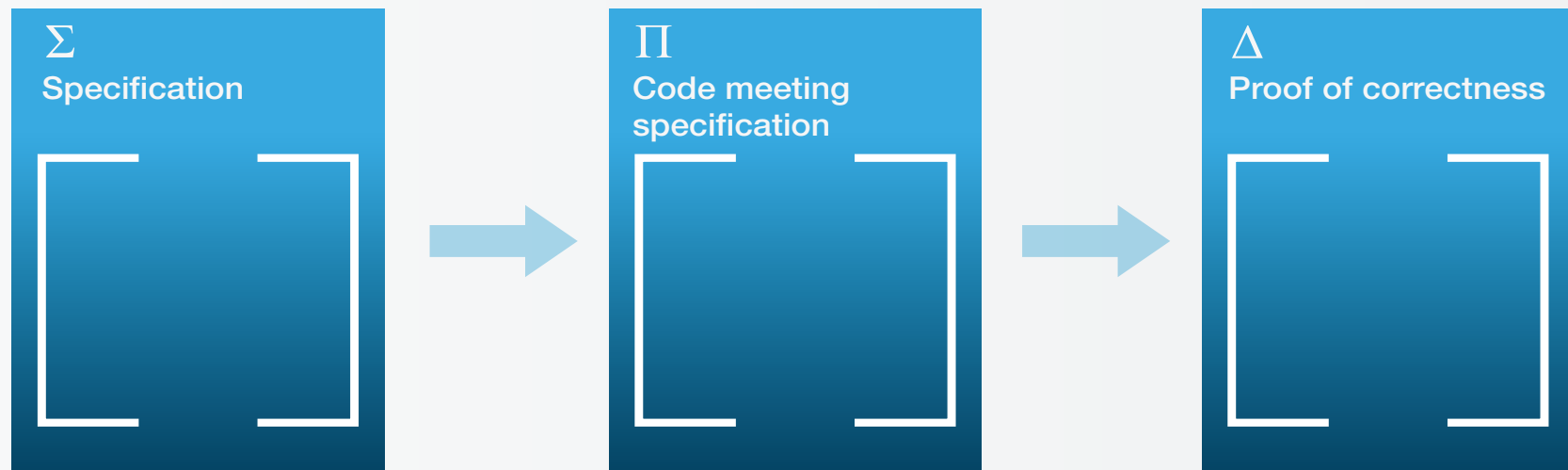
Specification



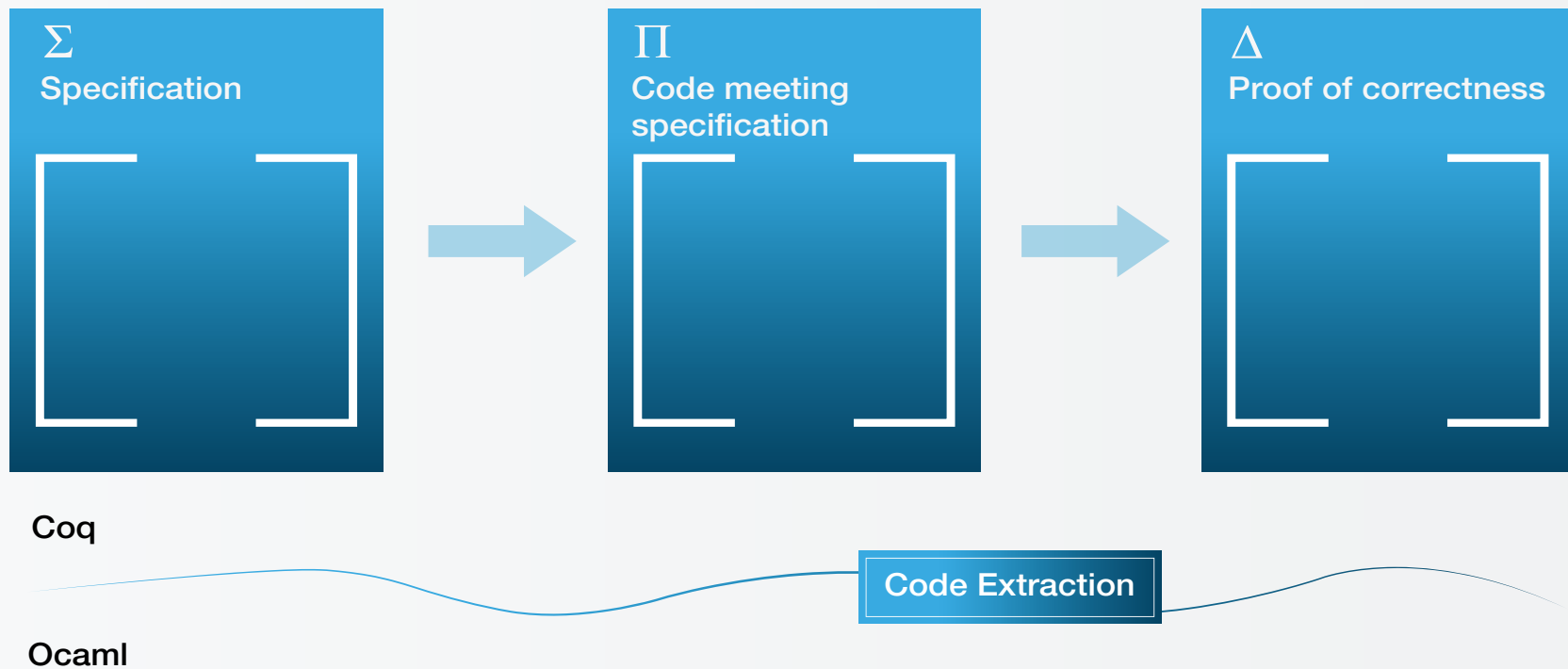
# VERIFICATION OF FUNCTIONAL CODE



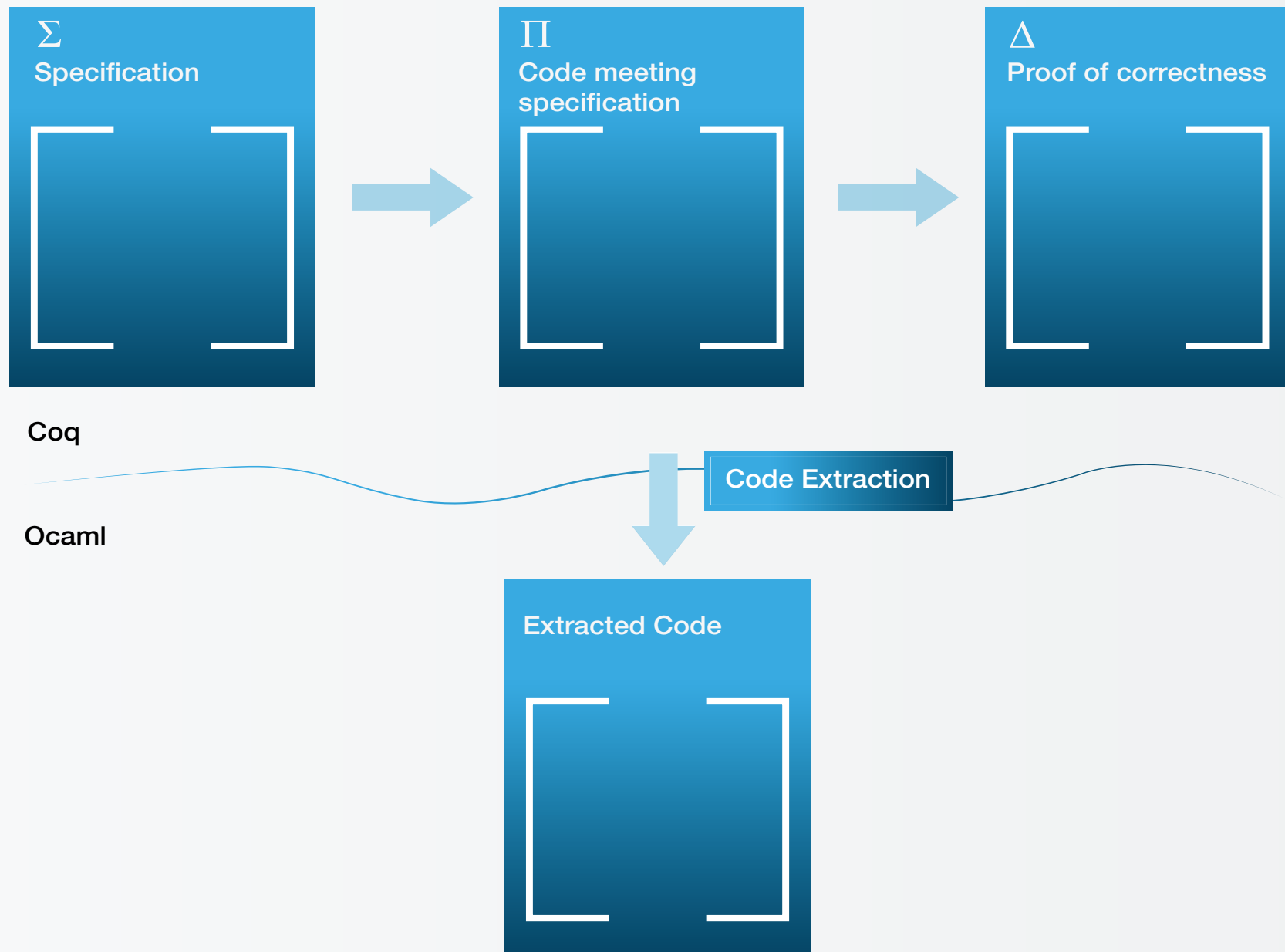
# VERIFICATION OF FUNCTIONAL CODE



# VERIFICATION OF FUNCTIONAL CODE



# VERIFICATION OF FUNCTIONAL CODE





The image shows the interior of a large, ornate cathedral. The architecture features high vaulted ceilings with intricate carvings and a series of large, arched windows on the upper level. The walls are covered in religious murals and frescoes, including a large scene on the left and another on the right. The floor is made of light and dark stone tiles in a diamond pattern. Rows of dark wooden pews with red upholstered seats are arranged on either side of a central aisle, leading towards the altar at the far end. A bright, glowing light emanates from the altar area. Overlaid in the center of the image is a white rectangular box with a blue border containing the text "THANK YOU!" in bold blue capital letters.

**THANK YOU!**