Turing machines and Computability, I

Construct explicit Turing machines for the following functions:

1. $\mathsf{Neg} : \{0,1\}^* \to \{0,1\}^*$, where $\mathsf{Neg}(x)$ is obtained from x by replacing the symbols 0 by 1 and 1 by 0.

2. $\mathsf{Even} : \{0,1\}^* \to \{0,1\}$, accepting a word x iff x has an even number of symbols.

3. (Unary addition) $\mathsf{Add} : \{0,1\}^* \to \{1\}^*$ mapping

$$\underbrace{1\ldots1}_{n \text{ times}}0\underbrace{1\ldots1}_{m \text{ times}} \longmapsto \underbrace{1\ldots1}_{n+m \text{ times}} .$$

4. (Huiswerk) $\mathsf{Inv} : \{0,1\}^* \to \{0,1\}^*$ rewriting a word x in reverse order.

Describe on the level of implementation:

5. Simulate any TM by a TM with the tape alphabet $\{0,1,\sqcup\}$.

6. Invent a definition of a 2-dimensional Turing machine and show its equivalence with a standard TM.

For the rest of your life you may use Church-Turing Thesis.

7. Show that the class of Turing recognizable (r.e.) languages is closed under union, intersection, concatenation.

8. Prove: an infinite subset of $\mathbf{N}$ is decidable iff there is a TM enumerating it in strictly increasing order.

9. Show: there is a language which is not r.e. Give an example.

10. Is there a language L such that neither L nor the complement of L is r.e.? (*) Give an example.[1]

11. There is a partial computable function $f : \mathbf{N} \to \mathbf{N}$ which has no total computable extension. (Try to play with the universal function.)

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[1] (*) means 'a more difficult problem'.