Recursion Theory

Joost J. Joosten

Institute for Logic Language and Computation

University of Amsterdam

Plantage Muidergracht 24

1018 TV Amsterdam

Room P 3.26, +31 20 5256095

jjoosten@phil.uu.nl

`www.phil.uu.nl/~jjoosten`

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- We shall study some notions but the emphasis will not be on the computational models themselves

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- We are building up a repertoire...