Final exam; practice version
Recursion Theory

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Practice exam

1. Let f be a computable function. Determine for each of the following implications if they are true or false. If true, give a very short proof, and if false, give a counterexample and a very short proof that, indeed, it is a counter example.

   (a) X is computable $\Rightarrow f(X)$ is computable

   (b) X is c.e. $\Rightarrow f^{-1}(X)$ is c.e.

   (c) $f(X)$ is computable $\Rightarrow X$ is c.e.

   (d) X is computable $\Rightarrow f^{-1}(X)$ is computable

2. Let C be a creative set and f a creative function for f. Find a different function f' which is also creative for C.

3. Prove that $A \in \Sigma^0_{n+1} \Leftrightarrow A$ is c.e. in $\emptyset^{(n)}$.

4. We call $R(x, y)$ a universal computable relation whenever it satisfies the following property. $R(x, y)$ is computable, and if $S(y)$ is a computable relation, then there is a natural number k such that $S(y)$ is true if and only if $R(k, y)$ is true.

   (a) Show that there exists no universal computable relation. (Hint: employ diogonalization.)

   (b) Use the previous exercise to show that $\{\mathsf{gn}(\varphi) \mid T \vdash \varphi\}$ and $\{\mathsf{gn}(\varphi) \mid T \vdash \neg\varphi\}$ are computably inseparable whenever T is a consistent theory extending Robinson's Arithmetic. (Hint: use representability of the computable relations in Q.)

5. (Separation principle for Π^0_1-sets.)

   Let A, B be disjoint Π^0_1-sets. Prove that there exists a computable relation C such that $A \subseteq C$ and $C \cap B = \emptyset$. (Hint:use the Reduction Principle for Σ^0_1-sets)

6. Describe why it is so that there exists some number e such that for all $X \subseteq \mathbb{N}$ we have that $W_e^X = X'$.