AUTOMATED THEOREM PROVING

Resolution in first-order logic

Exercise 1. Let $\theta_1 = \{a/x, f(z)/y, y/z\}$ and $\theta_2 = \{b/x, z/y, g(x)/z\}$. Find $\theta_1 \circ \theta_2$.

Exercise 2. If two or more literals with the same sign of a clause ϕ have a most general unifier σ, then $\phi\sigma$ is called a factor of ϕ. Give the factors of the following clauses:

(1) $P(x) \vee Q(y) \vee P(f(x))$.

(2) $P(x) \vee P(c) \vee Q(f(x)) \vee Q(f(c))$.

(3) $P(x, y) \vee P(c, f(c))$.

(4) $P(c) \vee P(d) \vee Q(x, y)$.

(5) $P(x) \vee P(f(y)) \vee Q(x, y)$.

Exercise 3. A clause ϕ_1 subsumes a clause ϕ_2, if there is a substitution σ such that $\mathrm{Mb}(\phi_1)\,\sigma \subseteq \mathrm{Mb}(\phi_2)$. Then, determine whether ϕ_1 subsumes ϕ_2 in each of the following cases:

(a) $\phi_1 = P(x, y) \vee Q(z)$, $\phi_2 = Q(c) \vee P(d, d) \vee R(u)$.

(b) $\phi_1 = P(x, y) \vee R(y, x)$, $\phi_2 = P(c, y) \vee R(z, d)$.

(c) $\phi_1 = \neg P(x) \vee P(f(x))$, $\phi_2 = \neg P(x) \vee P(f(f(x)))$.

Exercise 4. Determine whether each of the following sets is unifiable by using the unification algorithm.

(1) $\{Q(c), Q(d)\}$.

(2) $\{Q(c, x), Q(c, c)\}$.

(3) $\{P(x, y, z), P(y, z, y)\}$.

(4) $\{Q(c, x, f(x)), Q(c, y, y)\}$

(5) $\{Q(x, y, z), Q(u, h(v, v), u)\}$.

(6) $\{P(x_1, g(x_1), x_2, h(x_1, x_2), x_3, k(x_1, x_2, x_3)), P(y_1, y_2, e(y_2), y_3, f(y_2, y_3), y_4)\}$.

Exercise 5. Find all possible resolvents of the following pairs of clauses:

(1) $\phi_1 = \neg P(x) \vee Q(x, b)$, $\phi_2 = P(a) \vee Q(a, b)$.

(2) $\phi_1 = \neg P(x) \vee Q(x, x)$, $\phi_2 = \neg Q(a, f(a))$.

(3) $\phi_1 = \neg P(v, z, v) \vee P(w, z, w)$, $\phi_2 = P(w, h(x, x), w)$.

(4) $\phi_1 = \neg P(x, y, u) \vee \neg P(y, z, v) \vee \neg P(x, v, w) \vee P(u, z, w)$, $\phi_2 = P(g(x, y), x, y)$.

Exercise 6. Prove by resolution that the Skolem formulas α_Φ associated with the following sets of formulas are unsatisfiable:

(a) $\Phi = \{\neg P(x) \vee Q(f(x), x), P(g(b)), \neg Q(y, z)\}$.

(b) $\Phi = \{P(x), Q(x, f(x)) \vee \neg P(x), \neg Q(g(y), z)\}$.

Exercise 7. Prove by resolution that the formula $\exists x \forall y R(x, y) \rightarrow \forall y \exists x R(x, y)$ is a tautology.

Exercise 8. Consider the following assertions:

(1) ‘’The custom officials search everyone who enters the country and is not a VIP‘’.

(2) “Some drug pushers enter the country and they are only searched by drug pushers”.

(3) “No drug pusher is a VIP”.

(a) Formalize (1)-(3) as first-order formulas.

(b) Prove by resolution that from the assertions (1)-(3) it follows that “some of the custom officials are drug pushers”.