AUTOMATED THEOREM PROVING

Propositional Logic

Exercise 1. Prove by resolution that the following sets of formulas are unsatisfiable:

(a)
$$\Phi = \{P \lor Q \lor R, \neg P \lor R, \neg Q, \neg R\}.$$

(b)
$$\Phi = \{P \vee Q, \neg Q \vee R, \neg P \vee Q, \neg R\}.$$

Exercise 2. If the congress refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the president of the company resigns. Suppose that the congress refuses to act and the strike is over. Then, prove by resolution that the strike has lasted more than one year.

<u>Exercise 3</u>. Suppose that we can perform the following chemical reactions:

$$MgO + H_2 \rightarrow Mg + H_2O$$

 $C + O_2 \rightarrow CO_2$
 $CO_2 + H_2O \rightarrow H_2CO_3$

Suppose we have some quantities of MgO, H_2, O_2 y C. Then, prove by resolution that we can obtain H_2CO_3 (carbonic acid).

Exercise 4. Apply the Davis-Putnam rules to decide whether the following formulas are satisfiable.

- (1) $(\neg P \lor Q) \land \neg Q \land P$.
- (2) $P \wedge Q \wedge R$.
- (3) $(P \lor Q) \land \neg Q$.
- $(4) (P \vee Q) \wedge (R \vee Q) \wedge \neg R \wedge \neg Q.$
- (5) $(P \vee Q) \wedge (\neg P \vee Q) \wedge R$.
- (6) $(P \lor Q) \land (\neg P \lor Q) \land (\neg R \lor \neg Q) \land (R \lor \neg Q)$.

Exercise 5. By using the Davis-Putnam rules, decide whether the following sentence is contradictory: "if the congress refuses to enact new laws, the strike will not be over unless it lasts more than one year and the president of the company resigns, and if the congress enacts new laws or the strike is not over then the strike will last more than one year".

Exercise 6. Prove that the Davis-Putnam rules preserve satisfiability.