

Modal Logic

Exercise 1

Cory Knapp

Claim. $\vdash \Box A \rightarrow \Box(\Box(A \wedge \Box A) \rightarrow (A \wedge \Box A))$.

Proof.

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| 1. $(A \wedge (\Box A \wedge \Box \Box A) \rightarrow A \wedge \Box A) \rightarrow ((\Box(A \wedge \Box A) \rightarrow \Box A \wedge \Box \Box A) \rightarrow ((A \wedge \Box(A \wedge \Box A)) \rightarrow A \wedge \Box A))$ | Tautology |
| 2. $A \wedge (\Box A \wedge \Box \Box A) \rightarrow (A \wedge \Box A)$ | Tautology |
| 3. $(\Box(A \wedge \Box A) \rightarrow \Box A \wedge \Box \Box A) \rightarrow ((A \wedge \Box(A \wedge \Box A)) \rightarrow A \wedge \Box A)$ | MP, 1, 2 |
| 4. $\Box(A \wedge \Box A) \rightarrow \Box A \wedge \Box \Box A$ | Lemma |
| 5. $(A \wedge \Box(A \wedge \Box A)) \rightarrow A \wedge \Box A$ | MP, 3, 4 |
| 6. $((A \wedge \Box(A \wedge \Box A)) \rightarrow A \wedge \Box A) \rightarrow (A \rightarrow (\Box(A \wedge \Box A) \rightarrow A \wedge \Box A))$ | Tautology |
| 7. $A \rightarrow (\Box(A \wedge \Box A) \rightarrow A \wedge \Box A)$ | MP, 5, 6 |
| 8. $\Box(A \rightarrow (\Box(A \wedge \Box A) \rightarrow A \wedge \Box A))$ | Necessitation |
| 9. $\Box(A \rightarrow (\Box(A \wedge \Box A) \rightarrow A \wedge \Box A)) \rightarrow (\Box A \rightarrow \Box(\Box(A \wedge \Box A) \rightarrow A \wedge \Box A))$ | Distributivity |
| 10. $\Box A \rightarrow \Box(\Box(A \wedge \Box A) \rightarrow A \wedge \Box A)$ | MP, 10, 9 |

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