MODAL LOGIC
EXERSICES-1

RAMON JANSANA

1.

Let the model with diagram

$$\circlearrowleft \overset{p}{1} \underset{\longrightarrow}{\overset{q}{\longleftarrow}} \overset{q}{2}$$
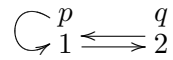
In which points the following formulas are true?

1. $\Box p \to \Box\Box p$
2. $\neg\Box p$
3. $p \to \Diamond\Box p$
4. $\neg\Box q \to \Box\neg p$
5. $\Diamond q \to \neg\Diamond q$

2.

Let $\mathcal{M} = \langle W, R, V \rangle$ be the model such that

$W = \{1, 2, 3, 4\}$,
$R = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle, \langle 4, 2\rangle\}$
$V(p) = \{1, 3\}, V(q) = \{1, 2\}$

(a) Draw the model.
(b) In which points the following formulas are true?

1. $\Box q$,
2. $\Box\neg(p \to \neg q)$,
3. $\Box(p \vee q) \vee \Diamond(p \wedge q)$,
4. $\Diamond\Box(p \vee q)$,
5. $\Box p \wedge \Diamond q$.

(c) Which of the following formulas are valid in $\mathcal{M}$?

1. $\Diamond\Box p \vee \Diamond\Diamond\Box p$,
2. $\Box p \to \neg p$,
3. $(p \to \Diamond p) \wedge (q \to \Diamond q)$,
4. $\Diamond(p \vee \neg p) \to \Box(p \vee \neg q)$.

3.

Show that the formulas
1. $\Box(\varphi \vee \psi) \rightarrow (\Box\varphi \vee \Diamond\psi)$
2. $(\Box\varphi \wedge \Diamond\psi) \rightarrow \Diamond(\varphi \wedge \psi)$

are valid on every frame.

4.

Prove that if a formula φ is valid on a frame $\mathcal{F}$, then every susbstitution instance of φ is valid on $\mathcal{F}$.

5.

Prove that for every normal modal logic L and all formulas φ, ψ,

$$\Diamond(\varphi \wedge \psi) \rightarrow (\Diamond\varphi \wedge \Diamond\psi) \in L.$$

You can assume (1), (2) and (3) of Proposition 5 already proved.