# Introducció a la lògica 2015–2016, (Code 360906)

## Practice midterm exam

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## Exercise 1

Give the definition of $\Gamma \models \varphi$.

## Exercise 2

Prove the transitivity of logical consequence. That is, prove that if $\varphi \models \psi$ and $\psi \models \chi$, then also $\varphi \models \chi$.

## Exercise 3

By $\bot$ we denote a logical constant that is always false. Thus, for any assignment (assignació) $v$ we have that $v(\bot) = 0$.

1. Write a formula $\varphi$ that only (at most) uses the connective $\rightarrow$ and the logical constant $\bot$ so the $\varphi \equiv \neg p$;
2. Write a formula $\varphi$ that only (at most) uses the connective $\rightarrow$ and the logical constant $\bot$ so the $\varphi \equiv p \wedge q$;
3. Write a formula $\varphi$ that only (at most) uses the connective $\rightarrow$ and the logical constant $\bot$ so the $\varphi \equiv p \vee q$;
4. Write a formula $\varphi$ that only (at most) uses the connective $\rightarrow$ and the logical constant $\bot$ so the $\varphi \equiv p \rightarrow q$;
5. Write a formula $\varphi$ that only (at most) uses the connective $\rightarrow$ and the logical constant $\bot$ so the $\varphi \equiv p \leftrightarrow q$;

In each case, prove that the formula you give is indeed logically equivalent.

Next, and to conclude this exercise, reason to the effect that each propositional formula is logically equivalent (classically) to a formula using at most the connective $\rightarrow$ and the constant $\bot$.

## Exercise 4

1. How many non-equivalent propositional logical formulas do there exist using only one propositional variable;
2. How many non-equivalent propositional logical formulas do there exist using only two propositional variables;
3. How many non-equivalent propositional logical formulas do there exist using only $n$ propositional variables;
   (Here, $n$ is an arbitrary natural number.)

## Exercise 5

1. How many non-equivalent propositional logical formulas do there exist using only one free variable and only the connective $\rightarrow$;
2. How many non-equivalent propositional logical formulas do there exist using only two free variables and only the connective $\rightarrow$;

## Exercise 6

Prove that $((p \rightarrow q) \rightarrow p) \rightarrow p$ is a tautology.

## Exercise 7

Mention the main philosophical and methodological choices that we made to end up with classical propositional logic.