Introducció a la lògica 2016–2017, (Code 360906)

Practice first partial exam

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October 27, 2016

About this document

Carlos Ibañez, one of the students, took this exam and edited his answers into the LATEX source of the exam. Next, Joost made some corrections and comments into the document. Joost's corrections are in red. So, the exam is written by Joost, the answers by Carlos and the corrections (in red) by Joost. I hope that this document is useful to the other students too.

What to study

Recall that at the end of the slides of the first lecture there is an overview of the material that is included. So please, download those slides from my webpage and look at the last page where you can find the overview. What do you need to prepare for the first midterm exam? Well, look at the scheme. I think it is Sections 1 and 2 of Chapters 6 and 7. Then the entire chapters 8 and 9 and finally Sections 1 and 2 of Chapters 1 and 2.

The best way to prepare your midterm is to make tons of exercises. Each chapter of the book comes with a selection of exercises. The more you make, the better you will prepared for the exam. Apart from that, I have prepared this midterm exam for you. Moreover, you can look at my webpage to the previous year where there are some further practice midterm exams.

Exercise 0

We consider two sets A and B given by $A := \{\{0\}, 1, 2\}$ and $B := \{0, 1, \{2\}\}$. Compute the following:

1. $A \cap B = \{1\}$

2. $A \cup B = \{\{0\}, 0, 1, 2\ ,\ \{2\}\}$

3. $A \setminus B = \{\{0\}, 2\}$

4. $B \setminus A = \{0, \{2\}\}$

Exercise 1

Give ten different formulas that are logically equivalent to p. However, you are not allowed to use more than three connectives per formula.

1. p
2. $p \wedge p$
3. $(p \wedge p) \wedge p$
4. $p \wedge (p \wedge p)$
5. $p \vee p$
6. $\neg p(\neg p)$ **This is not correct syntax. What you can say is of course $\neg\neg p$.**
7. $p \wedge q$ **NO, this formula is not equivalent to the formula p since I can find a valuation v so that $v(p) \neq v(p \wedge q)$. Namely $v(p) := 1$ and $v(q) := 0$.**
8. $(p \vee q, \neg q)$ **This is not a formula!!!! Probably you wanted to write the formula $(p \vee q) \wedge \neg q$ which is correct!!!!**
9. **There are still two more formulas missing!!!**

Exercise 2

In the following cases decide if the statement is true or false. In case the statement is true, prove it using a reasoning or a truth-table or show that is false by exhibiting a valuation.

1. $\{p \vee q, q \to p\} \models p \to q$; Per que **Your reasoning was correct however, you forgot to say false which we denote shortly by $\not\models$. Thus, you should have written**
   $\{p \vee q, q \to p\} \not\models p \to q$; Per que

1	1	1
1	**1**	**0**
1	0	1
0	1	1

The table should be unambiguous to read. You can achieve that as follows:

p	**q**	**p∨ q**	**q→ p**	**p→ q**
1	**1**	**1**	**1**	**1**
1	**0**	**1**	**1**	**0**
0	**1**	**1**	**0**	**1**
0	**0**	**0**	**1**	**1**

2. $\{p \rightarrow q, \neg(q \rightarrow r)\} \models \neg p$; Per que

1	0	0
1	**1**	**0**
0	0	0
0	0	0
1	0	1
1	1	1
1	0	1
1	0	1

   As before, your table should be 'decorated' and moreover, the argument establishes $\not\models$! However, I do not see what enumeration you have used since the refuting assignment v should be so that $v(p) = v(q) = 1$ and $v(r) = 0$.

3. $\{p \vee (q \vee r), p \rightarrow q, r \rightarrow q\} \models (p \rightarrow q) \rightarrow r$; Per que

1	1	1	1
1	**1**	**1**	**0**
1	0	0	1
1	0	1	0
1	1	1	1
1	**1**	**1**	**0**
1	1	0	1
0	1	1	0

   The same comment from the previous exercise applies to here. In addition, I would like to add that in order to show that $\not\models$, it suffices to give just one assignment that validates the antecedent but does not validate the consequent.

4. $\{\neg p \vee q, (p \rightarrow q) \rightarrow p, \neg(\neg q \rightarrow r)\} \models s$;
   As we discussed in class, if the antecedent is not satisfiable, then anything will be a logical consequence of it.

Exercise 3

Tell of the following statements if they are true or false and motivate your answer.

1. If φ is a tautology and ψ a contingency, then

$$\{\varphi\} \models \psi.(\not\models)$$

Exactly, since ψ is a contingency, there is some assignment v so that $v(\psi) = 0$. Further, since φ is a tautology, we have $v(\varphi) = 1$. Thus, the statement is true.

2. If φ is a contingency and ψ a contingency, then

$$\{\varphi\} \models \psi.(\not\models)o(\models)$$

   So the correct answer should be that the statement is false. As you point out, we can get $\models$ (e.g. by taking $\varphi = \psi = p$) or we can get $\not\models$ (e.g. by taking $\varphi = p$ and $\psi = q$)

3. If φ is a contingency and ψ a tautology, then

$$\{\varphi\} \models \psi.(\models)$$

   That is, the statement is true.

4. If φ is a contingency and ψ a contradiction, then

$$\{\varphi\} \models \psi.(\not\models)$$

   That is, the statement is false.

5. If φ is a contradiction and ψ a contingency, then

$$\{\varphi\} \models \psi.(\models)$$

   That is, the statement is true.

Exercise 4

For each of the following formulas, find equivalent formulations that only use the connectives $\vee$ and $\neg$

1. $p \to (r \to s) \equiv \neg p \vee (s \vee \neg r)$
2. $(p \to q) \to (r \to s) \equiv \neg\, (\neg\, p \vee q) \vee (\neg r \vee s)$
3. $p \wedge (q \to r) \equiv \neg\, \vee \neg(\neg q \vee r)$
   This is of course not syntactically correct. I guess you meant

$$p \wedge (q \to r) \quad \equiv \quad \neg(\neg p \vee \neg(\neg q \vee r))$$

4. $(p \vee \neg r) \vee \neg(p \vee q) \equiv \neg\, (\neg p \to \neg r) \to \neg(\neg p \to q)$
   This was a trick question: the formula was already in the required form, haha.
5. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
   That is the first step, next you will have to eliminate the implications and the conjunction.

Exercise 5

Give the genealogical tree of the following formulas.

1. $\neg p \to (q \wedge (r \vee s))$;
2. $\neg((p \to (q \wedge r)) \vee s$;
3. $\neg(((p \to (q \wedge r)) \vee s)$;
4. $\neg(p \to q) \wedge (r \vee s)$;
5. $\neg((p \to q) \wedge (r \vee s))$;
6. $\neg((p \to q) \wedge (r\vee) s)$;

Exercise 6

1. What is the definition of *propositional classical logical tautology*?
2. What are the differences between classical and intuitionistic logic? (This is not part of the exam material.)

Exercise 7

What are the roles of logic in philosophy?

Exercise 8

Tell of the following formulas if they are contingencies, tautologies, or contradictions? Motivate your answer.

Again, the answers that you give should come with a disambiguation of the tables...

1. $(\neg p \to p) \to p$;

1	**1**
1	**1**
0	**1**
0	**1**

If there is just one variable, then you only need to consider 2 assignations.

Aquesta formula es una tautologia, doncs totes les assignacions de la seva taula de veritat son verdaderes.

2. $(p \to q) \vee (q \to p)$;

1	**1**	1
0	**1**	1
1	**1**	0
1	**1**	1

Aquesta formula es una tautologia....

Again, you need to decorate your table. Moreover, we prefer to include a column for every subformula.

3. $(p \to q) \vee (q \to r) \vee (r \to p)$;

4. q;

5. $\neg p \wedge \neg(p \to q)$;

6. $((p \to q) \to p) \to p$.

I am very glad that you set aside the natural inertia to get acquainted with a new IT tool. As you see, working in Latex is very easy and the result looks so nice that it kind of pays of. Moreover, within science almost everybody uses it.