

Introducció a la lògica 2016–2017, (Code 360906)

Practice first partial exam

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Friday, October 14, 2016

What to study

Recall that at the end of the slides of the first lecture there is an overview of the material that is included. So please, download those slides from my webpage and look at the last page where you can find the overview. What do you need to prepare for the first midterm exam? Well, look at the scheme. I think it is Sections 1 and 2 of Chapters 6 and 7. Then the entire chapters 8 and 9 and finally Sections 1 and 2 of Chapters 1 and 2.

The best way to prepare your midterm is to make tons of exercises. Each chapter of the book comes with a selection of exercises. The more you make, the better you will be prepared for the exam. Apart from that, I have prepared this midterm exam for you. Moreover, you can look at my webpage to the previous year where there are some further practice midterm exams.

Exercise 0

We consider two sets A and B given by $A := \{\{0\}, 1, 2\}$ and $B := \{0, 1, \{2\}\}$. Compute the following:

1. $A \cap B =$
2. $A \cup B =$
3. $A \setminus B =$
4. $B \setminus A =$

Exercise 1

Give ten different formulas that are logically equivalent to p . However, you are not allowed to use more than three connectives per formula.

Exercise 2

In the following cases decide if the statement is true or false. In case the statement is true, prove it using a reasoning or a truth-table or show that is false by exhibiting a valuation.

1. $\{p \vee q, q \rightarrow p\} \models p \rightarrow q$;
2. $\{p \rightarrow q, \neg(q \rightarrow r)\} \models \neg p$
3. $\{p \vee (q \vee r), p \rightarrow q, r \rightarrow q\} \models (p \rightarrow q) \rightarrow r$;
4. $\{\neg p \vee q, (p \rightarrow q) \rightarrow p, \neg(\neg q \rightarrow r)\} \models s$;

Exercise 3

Tell of the following statements if they are true or false and motivate your answer.

1. If φ is a tautology and ψ a contingency, then

$$\{\varphi\} \models \psi.$$

2. If φ is a contingency and ψ a contingency, then

$$\{\varphi\} \models \psi.$$

3. If φ is a contingency and ψ a tautology, then

$$\{\varphi\} \models \psi.$$

4. If φ is a contingency and ψ a contradiction, then

$$\{\varphi\} \models \psi.$$

5. If φ is a contradiction and ψ a contingency, then

$$\{\varphi\} \models \psi.$$

Exercise 4

For each of the following formulas, find equivalent formulations that only use the connectives \vee and \neg

1. $p \rightarrow (r \rightarrow s)$;
2. $(p \rightarrow q) \rightarrow (r \rightarrow s)$;
3. $p \wedge (q \rightarrow r)$;
4. $(p \vee \neg r) \vee \neg(p \vee q)$;
5. $p \leftrightarrow q$.

Exercise 5

Give the genealogical tree of the following formulas.

1. $\neg p \rightarrow (q \wedge (r \vee s))$;
2. $\neg((p \rightarrow (q \wedge r)) \vee s)$;
3. $\neg(((p \rightarrow (q \wedge r)) \vee s)$;
4. $\neg(p \rightarrow q) \wedge (r \vee s)$;
5. $\neg((p \rightarrow q) \wedge (r \vee s))$;
6. $\neg((p \rightarrow q) \wedge (r \vee s))$;

Exercise 6

1. What is the definition of *propositional classical logical tautology*?
2. What are the differences between classical and intuitionistic logic? (This is not part of the exam material.)

Exercise 7

What are the roles of logic in philosophy?

Exercise 8

Tell of the following formulas if they are contingencies, tautologies, or contradictions? Motivate your answer.

1. $(\neg p \rightarrow p) \rightarrow p$;
2. $(p \rightarrow q) \vee (q \rightarrow p)$;
3. $(p \rightarrow q) \vee (q \rightarrow r) \vee (r \rightarrow p)$;
4. q ;
5. $\neg p \wedge \neg(p \rightarrow q)$;
6. $((p \rightarrow q) \rightarrow p) \rightarrow p$.