Lògica 2016–2017, (Code 360961)

Practice midterm exam

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About this exam

This is a practice midterm exam. How to prepare for the real exam? Well, make this exam and further, do exercises related to the material that we dealt with in class. You can look at the webpage to see what we have done. Furthermore, on my webpage, you will find slides at the first week where we have an overview of what material should be studied for this course. In particular, for the midterm you will have to know Chapter 6, Sections 1 and 2, Chapter 7, Sections 1 and 2, Chapters 8, 9 and 11, Chapter 1, Sections 1 and 2, Chapter 2, Sections 1 and 2, Chapter 3, Sections 1 – 5, Chapter 4 Sections 1 – 2, Chapter 12 and Chapter 13, Sections 1 – 4. Of course, in the real exam as in this practice exam not all material will be tested. Thus for example, the fact that equivalence relations are not in this practice exam is by no means a guarantee that they will not occur in the real midterm exam.

1 Exercise 1

Decide if the following statements are true or false. In case they are true, provide a reasoning to that extent, if they are false, give a counter-example.

1. If a relation R is irreflexive and symmetric it is transitive;
2. If a relation R is transitive and symmetric it is reflexive;
3. If a relation R is irreflexive and transitive it is anti-symmetric;
4. If a relation R is irreflexive and asymmetric it is transitive;

2 Exercise 2

Let $A := \{1, \varnothing, \{1, \varnothing\}, \{3, \varnothing\}\}$, let $B := \{\{2\}, \{\varnothing\}, \{\varnothing, 1, \varnothing\}, 1\}$ and let $C := \{1, \{2\}, \{3, \varnothing\}\}$. Compute the following:

1. $(A \cap B) \cup C$;
2. $A \cap (B \cup C)$;
3. $A \cap (B \cup C) - ((A \cap B) \cup C)$;
4. $A \cap (B \cap C)$;
5. $A \cup (B \cup C)$.

3 Exercise 3

In the following cases decide if the statement is true or false. In case the statement is true, prove it using a reasoning or a truth-table or show that is false by exhibiting a valuation.

1. $\{p \vee q, q \to p\} \models p \to q$;
2. $\{p \to q, \neg(q \to r)\} \models \neg p$
3. $\{p \vee (q \vee r), p \to q, r \to q\} \models (p \to q) \to r$;
4. $\{\neg p \vee q, (p \to q) \to p, \neg(\neg q \to r)\} \models s$;

4 Exercise 4

For each of the following formulas do the following:

A Give the genealogical tree corresponding to the following formulas (recall our conventions on bracket omissions);

B Point out the free occurrences of variables;

C Give the bound variables.

1. $\forall x(\neg P(x, y) \to \exists z S(x, y, z) \wedge R(z, z))$;
2. $\forall x(\neg P(x, y) \to \exists z(S(x, y, z) \wedge R(z, z)))$;
3. $\forall x \neg(P(x, y) \to \exists z S(x, y, z) \wedge R(z, z))$;
4. $\forall x \neg P(x, y) \to \exists z S(x, y, z) \wedge R(z, z)$.
5. $\forall x \neg(P(x, y) \to \exists z(S(x, y, z) \wedge R(z, z)))$;

5 Exercise 5

Exercises 3, (h)–(m) on Page 258 of the book.

6 Exercise 6

Consider the predicate logical language with two predicates C and M, two names/constants t and j and one binary predicate R. We consider the following

translation key:		
	Cx	x is a cat
	Mx	x is a mouse
	R x y	x chases y
	t	Tom
	j	Jerry

Translate the following sentences to our language of predicate logic using the given translation key.

1. Tom is a cat and Jerry is not a cat but a mouse;
2. Tom chases Jerry;
3. Any cat chases some mouse;
4. Some mouse chases any cat;
5. Any cat chases any mouse that does not chase any cat;
6. Any cat chases any mouse that does not chase some cat;
7. Any cat chases some mouse that does not chase some cat;
8. Some cat chases any mouse that does not chase some cat.

7 Exercise 7

In this exercise you are asked to provide functions on the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$.

1. Find a function that is injective but that is not a bijection; What are the domain, range and field of this function.
2. Find a function that is surjective but that is not a bijection; What are the domain, range and field of this function.
3. Find a function f that is a bijection but so that f is not the identity. What are the domain, range and field of this function.