EXERCISES

JJJ

1. Second batch of exercises

(1) This exercise is related to cut elimination in the following sense. Cuts are eliminated but to eliminate one cut, possibly we generate many more others. In the end however, we can eliminate them all.

There are two players, Player A and Player B. They start with a finite multiset X_0 of natural numbers, that is, $\mathsf{Set}(X_0) \subseteq \mathbb{N}$. Each round consists of the following. Player A takes out a number $x_i \in X_i$ of her choice so to get $A_i := X_i \setminus \{x_i\}$. Player B may then choose a number $y_i < x_i$ and a natural number n_i to form X_{i+1} by adding n_i many copies of y_i to A_i. In case $x_i = 0$, Player B will skip her turn so that $X_{i+1} := A_i$. Prove that for any strategies of A and B, there is some number m so that $X_m = \varnothing$.

(2) Recall the definitions for $s(A)$ the size of a formula A and $|A|$ the complexity of a formula A. We consider formulas that are built only from propositional variables, $\perp$ and $\to$.
   (a) Find a sequence of formulas $\{A_n\}_{n\in\omega}$ so that $|A_n| = n$ and with $s(A)$ 'as large as possible' (see below);
   (b) Give an exact expression of $s(A_n)$ in terms of n. Let us call this $f(n)$;
   (c) Prove that for any formula A we have $s(A) \leq f(|A|)$ (you can only do this if you have chosen your A_n right);
   (d) Prove that $s(A) \leq 2^{|A|+1}$.

(3) Given a formula A in propositional logic. Compute the number of possible subformulas of A. Explain your answer.

(4) Make Exercise 3.5.7.A from the book.