

Proof Theory, 2021

Joost J. Joosten

January 14, 2022

Exercises, Set 5

Exercise 0.1. *In this exercise you are asked to reflect on the sketch of completeness of predicate calculus as we have seen in class. Thus, you are asked to describe non-successful branches in proof-attempt trees of non-provable formulas and describe the term-model that arises from it.*

1. *Describe how a proof-attempt tree for $\Rightarrow \exists xPx \rightarrow \forall xPx$ gives rise to a term model.*
2. *Describe how a proof-attempt tree for $\Rightarrow \forall x\forall y(Rxy \rightarrow Ryx) \rightarrow \forall xRxx$ gives rise to an infinite term model. Of course, there will be many term-models described in our infinite proof-attempt tree. You are asked to describe just one infinite branch and the term model defined by that. Observe that our method fails to deliver the simplest one-point/empty-relation counter model!*

The following question is not part of the homework and is just there to tickle your curiosity. Can every possible counter model of a formula be obtained as a term-model of our proof-attempt tree? Correct answers will count as bonus points.

Exercise 0.2. *Prove the Substitution of Terms Lemma which is Lemma 3.5.2 in the book.*

Exercise 0.3. *For classical logic, provide the step in the proof of Cut Elimination for the case the cut formula is of the form $\exists xA$ and is active on both premisses.*