AUTOMATED THEOREM PROVING

First-order Logic

Exercise 1. Give a Skolem standard form for each of the following formulas:

(a) $\neg(\forall x Px \rightarrow \exists y \forall z Qyz)$,

(b) $\forall x(\neg Rxc \rightarrow \exists y(Ryg(x) \wedge \forall z(Rzg(x) \rightarrow Ryz))$,

(c) $\neg(\forall x Px \rightarrow \exists y Py)$.

Exercise 2. Prove by means of a counterexample that, in general, a formula ϕ of a first-order language is not equivalent to a Skolem form of ϕ.

Exercise 3. By using Herbrand's Theorem, show that the following formulas are unsatisfiable:

(a) $\forall x(Px \wedge \neg Pf(x))$,

(b) $\forall x \forall y \forall z(Px \wedge (Qxf(x) \vee \neg Px) \wedge \neg Qg(y)z)$.

Exercise 4. Explain where the proof of Herbrand's Theorem fails if we allow the identity in first-order formulas. Give a counterexample.

Exercise 5. Determine whether each of the following sets is unifiable by using the unification algorithm.

(1) $\{Q(c), Q(d)\}$,

(2) $\{Q(c, x), Q(c, c)\}$,

(3) $\{P(x, y, z), P(y, z, y)\}$,

(4) $\{Q(c, x, f(x)), Q(c, y, y)\}$,

(5) $\{Q(x, y, z), Q(u, h(v, v), u)\}$,

(6) $\{P(f(x, g(a, y)), h(x)), P(f(f(u, v), w), h(f(a, b)))\}$.

Exercise 6. If φ is a formula with the free variables $x_1, \dots x_n$, we denote by $\forall\varphi$ the formula $\forall x_1 \dots \forall x_n \varphi$. Prove that if a clause φ is a resolvent of two clauses φ_1 and φ_2, then $\forall\varphi$ is a logic consequence of $\{\forall\varphi_1, \forall\varphi_2\}$.

Exercise 7. Find all resolvents of the following two clauses:

$\varphi_1 = \neg P(x, y) \vee \neg P(f(a), g(u, b)) \vee Q(x, u)$,

$\varphi_2 = P(f(x), g(a, b)) \vee \neg Q(f(a), b) \vee \neg Q(a, b)$.

Exercise 8. Prove by resolution that the formula $\exists x \forall y R(x, y) \to \forall y \exists x R(x, y)$ is a tautology.

Exercise 9. Consider the following assertions:

(1) ‘’The custom officials search everyone who enters the country and is not a VIP‘’.

(2) “Some drug pushers enter the country and they are only searched by drug pushers”.

(3) “No drug pusher is a VIP”.

(a) Formalize (1)-(3) as first-order formulas. For this, use E(x) for “x enters the country”; P(x) for “x is a VIP”; C(x) for “x is a custom official”; S(x,y) for “x searchs y”; and D(x) for “x is a drug pusher”.

(b) Prove by resolution that from the assertions (1)-(3) it follows that “some of the custom officials are drug pushers”.

Exercise 10. By using the rules of Robinson and Wos, show that the formula $\exists x \exists y (Pdx \wedge Pey)$ is a logic consequence of the set of formulas $\{a = c, b = c, c = d, c = e, Pax \vee Pby\}$.